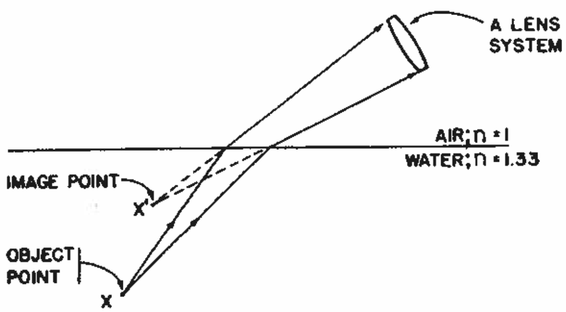
Melvin L Rubin, MD, MS. (1993). Optics for Clinicians. Florida: Triad Publishing Company

Thin flexible tubes of transparent material capture light at one end and keep it "trapped" inside by total reflection until it reaches the other end, where it is allowed to exit. The beauty of these "light pipes" is they can be bent into unusual shapes and the light will follow the bends, seemingly in defiance of the principle of *linear* propagation of light. Not only can the light emitted at the end of such a "pipe" be used as an illumination source for dark recesses, say, of the inside surface of the stomach; if properly constructed, *optical* fiber bundles (created by thousands of very fine, perfectly aligned, closely packed tubules) can allow sharp visualization and photography of these same, hitherto unseeable areas. New endoscopic applications for these fiber optic bundles are continually being discovered.

You should also know that the visual cells (rod and cone) outer segments "trap" light via this same mechanism of total internal reflection and thus permit more efficient utilization of light entering the eye.

Prismatic Deviation

So far, we have explored the refraction of a *single* light ray by a plane surface separated by two different media. Now let us follow a number of these rays which leave an object point.

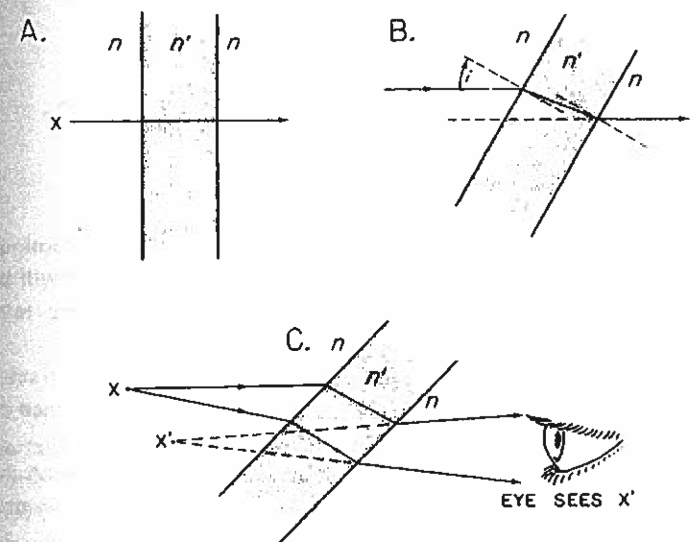


Object point X is immersed in water. The light rays from X are shown; each is bent appropriately (Snell's Law) as it leaves the water. These rays will seem to emanate from point X' (in a straight line path)

to any optical system (camera lens or a human eye) which intercepts them. Point X' will seem closer and not as deep in the water as the original point X, and accounts for the "displaced" position of objects which are under water. The greater the viewing angle θ makes with the water, the greater the apparent displacement. But, even if the view is directly perpendicular to the surface, the image point X' will still seem closer than the object really is. (You should now be able to diagram this yourself.)

This is our first exposure to "displacement" caused by a plane refracting surface. It leads us to consider refraction and displacement by a two surfaced plane-parallel, such as a slab of glass with parallel faces.

Refraction by two plane-parallel surfaces:

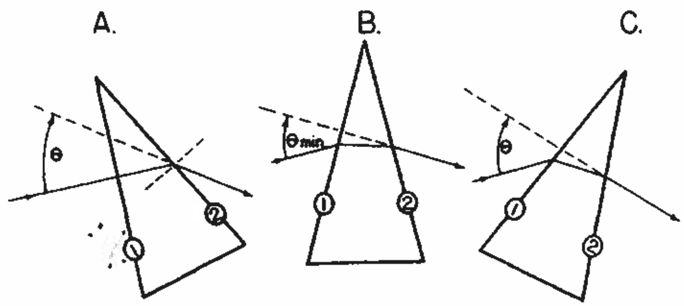


A) If the sheet of glass has 2 parallel faces and a ray strikes along the "normal", there is no deviation of this ray.

B) If the glass plate is tilted so that the "normal" makes an angle i to the incoming ray, the ray will be bent towards the normal inside the glass and then away from the new normal as it exits again.

C) To an eye looking through the glass at rays from point X, the object will appear closer and displaced, as if it were situated at X'. *This* is prismatic deviation (or displacement). To an observing eye, this type of deviation will appear to be much more pronounced for *near* objects than for distant ones.

Refraction by two plane *non*-parallel surfaces: If the sides of the glass are not parallel, we have a "prism"; there will always be a deviation imparted to any incoming ray.



A) If a ray strikes the first face along its normal, it will continue without angulation until it strikes the second face, when it will be bent away from *its* surface normal; as shown above, the total angular displacement is θ.

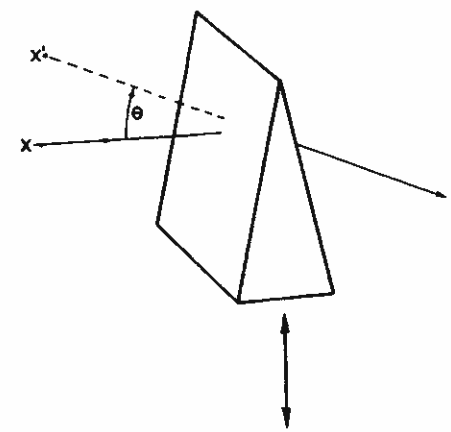
B) θ will be *minimal* for any given prism when the ray travels symmetrically through the prism, so that the angle of incidence on face 1 is equal to the angle of refraction from face 2.

C) If the prism is pivoted further so that the ray proceeds through the prism asymmetrically again, the deviation of the ray will *increase* from the minimum shown in B.

In general, then, the capability or effectivity of a prism in bending rays is indicated by its "power", which is usually measured in prism diopters. The "power" is that which causes the *"minimum"* angular deviation, though we should now realize that the prism's power is *not* absolutely fixed and varies a little as you tilt or pivot it. (The variation

with tilt is greater with prisms of higher power. From this you can draw the following clinical conclusion: if you desire an accurate measurement of a phoria or a muscle weakness, especially if you are using higher powered prisms, you must hold the testing prisms squarely — not tilted — in front of a patient's eye.)

Aside from the effect of tilting the prism, for most practical purposes, you should consider the induced deviation for any clinical prism as *constant;* that is, (see figure below) if you sight an object X through the prism, there will be an immediate but fixed apparent displacement of that object to X'. If you continue to sight while moving the prism up and down along its base-apex dimension, that object will "stay put"; it will *not* appear to move.

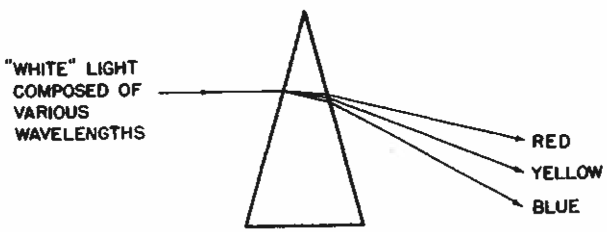


The fact that an object does not appear to move in this situation tells us that there is no *spherical* power incorporated along with the prism power; apparent "movement" *would* occur if there were refractive (vergence) power present. We will later learn how to "hand neutralize" unknown corrective lenses by using this "movement" technique.

Another point about prisms: when you look through one, you should also discern that distortion is introduced — straight lines may look curved and parallel lines may appear to diverge. Actually three *different* types of distortions are introduced by prisms and each has its own characteristic optical basis.* Sufficient for us is simply knowing that prisms induce distortion in addition to the object displacement.

In another short digression, we initially mentioned that the refractive index of any material varied not only with the material itself but with the *wavelength* of the incident light. Thus, if light composed of many wavelengths falls onto a prism such as that shown below, each of the component wavelengths is refracted a *different* amount; the shortest wavelength (blue light) is bent most, and red, the longest wavelength, the least. This is the chromatic dispersion you saw when you played with prisms in nursery school. It is an important occurrence and is the physical basis for the "chromatic aberration" of all optical systems.

Practical use is made of this "aberration" in an instrument called the spectroscope (for physico-chemical analyses) and clinically, in the *"Bichrome"* or "Duo-chrome" test, to be explained later.



PRISMATIC DISPERSION

The extensive foregoing discussion then, explains how prisms work and how Snell's Law governs ray deviation and object displacement. In summary, the basic prism "power" is derived from two factors: how steeply the flat faces of the prism abut and the index of refraction of the prism material itself. (The greater the index, the greater the overall power.)

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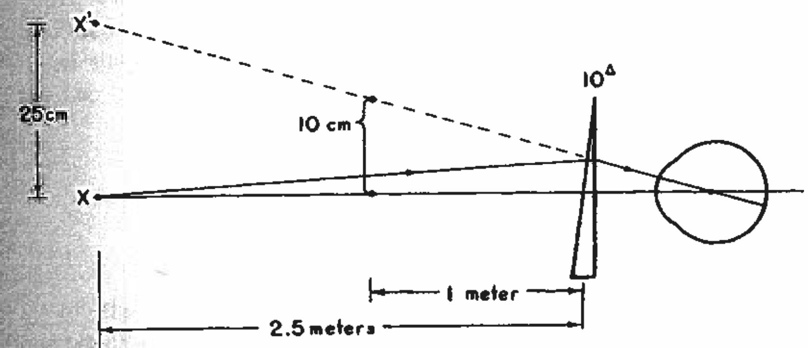
* (See Optics, by Ken Ogle, Thomas, 1968, p. 74-5)

Ophthalmic Prisms

Realize that in using prisms in ophthalmology, the eye must pivot (about its center of rotation) to follow the displaced images. Though this direction of displacement is always toward the prism's apex, *prism* positions are typically given (just to confuse you) by their *base* locations! A 10 prism diopter *base*-down prism would displace an image upward, so that it appears off-axis by 10 cm for each meter of distance, (or 25 cm for a distance 2.5 meters away). The eye must rotate upwards to fixate the displaced object. (See diagram.)



Ophthalmic prisms come in various forms:

1) single, loose prisms of varying powers.

2) bar prisms — a graded series of prisms mounted on a stick. Usually they are constructed so that when the bar is held vertically, they introduce a horizontal prismatic effect. Bars introducing vertical prisms are also available.

3) A "Risley" prism — which is of continuously variable power. This is constructed with two 15^{Δ} prisms mounted so that by turning a single knob, you rotate both prisms equally, but in opposite directions. This maintains a resultant prism activity in one primary direction. When the prisms are fully aligned (with their bases pointing in the

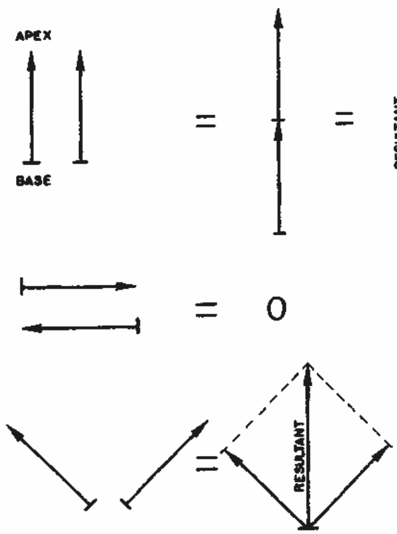
same direction), their two powers are additive and will yield 30^{Δ} (see A below). If they are rotated so that the apex of one falls on the base of the other, the resultant prism power is 0^{Δ}. (See B.) Partial rotations yield the "in-between" powers (see C). This tool is extremely handy and is supplied commercially on almost every type of refracting unit made for clinical purposes.

THE RISLEY PRISM

A.

B.

C.



4) Fresnel prisms — to be covered later.

We have now covered sufficient detail to understand the properties of refraction by plane surfaces and how Snell's Law supervises this activity.

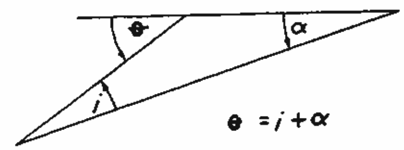
CURVED SURFACE REFRACTION

Let us now investigate how curved surfaces might act on light rays.

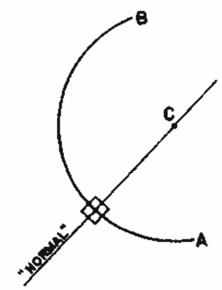
PLEA: Do not allow this one derivation to scare you away. Bear with me and do make an effort to go through it. It is *not* tough despite the use of a few mathematical symbols. Actually, you may find it is fun to learn exactly how your useful formula, $U + P = V$, is derived. However, those readers who find even simple mathematics abhorrent can proceed to skip over to page 71.

To see what happens at a curved surface, two points must be resurrected from your geometrical background:

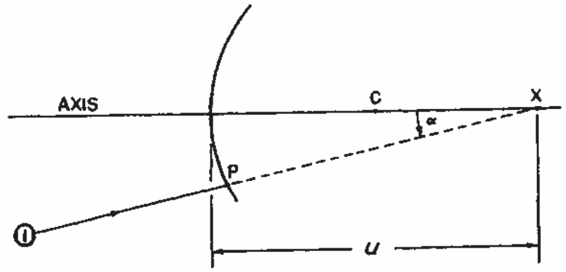
a) the *exterior* angle of any triangle equals the sum of the opposite two interior angles:



b) The "normal" (perpendicular) to a spherical surface is any line which passes through the center of curvature of that surface. If C is the center of curved arc AB, the line shown is considered "normal" to that surface:



Now, let us take a brief glimpse at refraction at the curved surface. Incoming ray 1 strikes point P on a spherical surface whose center is at C. (See next figure.)



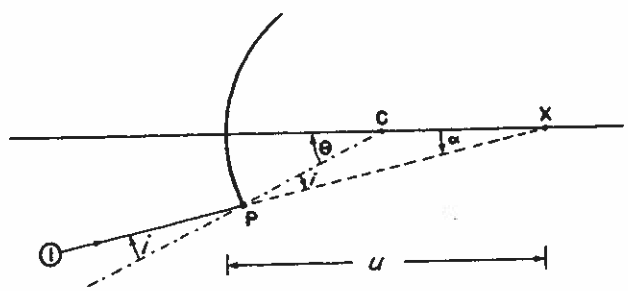
If this ray did not hit the surface, it would continue to travel and intersect the axis at point X. The distance to X (measured along the axis) is object distance u, and the angle this ray 1 makes with the axis is α.

To point P from C, we draw a line; this is "normal" to the surface. The "normal" makes angle θ at C, and incoming ray 1 makes angle i with this "normal". Angle i is the "angle of incidence"; it is equal to its *vertical angle* (created by two intersecting lines), shown below as one angle of triangle PCX.

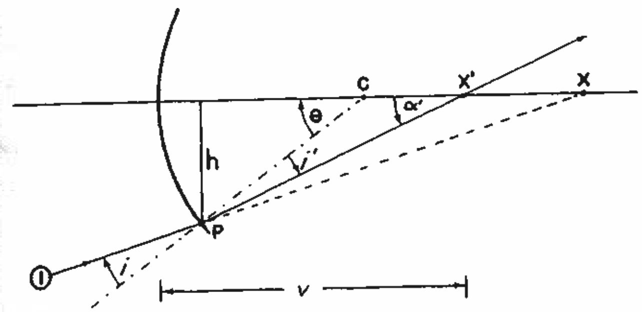
Since θ is an exterior angle of PCX,

$$\theta = \alpha + i$$

So, $$i = \theta - \alpha$$



Ray 1 will be refracted by the surface and bent toward the "normal"; the "angle of refraction" is now i' (next figure.)



This refracted ray will continue and intersect the axis at X' making angle α' with it, and at an image distance v from the spherical surface.

θ is also an exterior angle of triangle PCX', so

$$\theta = i' + \alpha'$$

and $$i' = \theta - \alpha'$$

We have shown, then, that

$$i = \theta - \alpha$$
$$i' = \theta - \alpha'$$

In our simplification of Snell's Law for small angles,

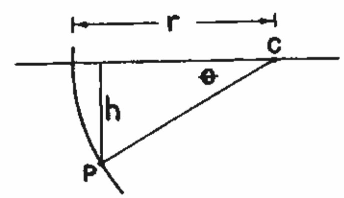
$$n\, i = n'\, i'$$

So, substituting for i and i',

$$n\, i = n\,(\theta - \alpha) = n'\, i' = n'\,(\theta - \alpha')$$

We will call the following, *FORMULA 1:*

$$n\,(\theta - \alpha) = n'\,(\theta - \alpha')$$



The distance from C to the curved surface is, of course, the radius r of that surface. Assume point P is at a distance h from the axis. When P is relatively close, that is, when h is short, θ will be a small angle, and h will just about be equal to the length of the arc subtended by θ. So, in radians,

$$\theta = \frac{h}{r}.$$

From the previous set of diagrams, and using the same reasoning, you can see that

$$\alpha' = \frac{h}{v}$$

and $$\alpha = \frac{h}{u}$$

To eliminate consideration of the actual angles themselves, (θ, α', and α), we can substitute the equivalent expressions for the angles into FORMULA 1:

$$n(\theta - \alpha) = n'(\theta - \alpha')$$

$$n\left(\frac{h}{r} - \frac{h}{u}\right) = n'\left(\frac{h}{r} - \frac{h}{v}\right)$$

Factoring out h,

$$h\,n\left(\frac{1}{r} - \frac{1}{u}\right) = h\,n'\left(\frac{1}{r} - \frac{1}{v}\right)$$

Dividing by h,

$$n\left(\frac{1}{r} - \frac{1}{u}\right) = n'\left(\frac{1}{r} - \frac{1}{v}\right)$$

Multiplying this out,

$$\frac{n}{r} - \frac{n}{u} = \frac{n'}{r} - \frac{n'}{v}$$

Regrouping,

$$\frac{n'}{v} - \frac{n}{u} = \frac{n'}{r} - \frac{n}{r}$$

Since $$\frac{n'}{r} - \frac{n}{r} = \frac{n' - n}{r}$$

Then $$\frac{n'}{v} - \frac{n}{u} = \frac{n' - n}{r}$$

Or $$\frac{n}{u} + \frac{n' - n}{r} = \frac{n'}{v}$$

EUREKA! This is the general relationship for a single spherical refracting surface. It ties together object vergence $(\frac{n}{u})$, image vergence $(\frac{n'}{v})$, and the refracting power of the surface $(\frac{n' - n}{r})$. Starting to look like something familiar? It should!

$$\frac{n}{u} = U$$

$$\frac{n'}{v} = V$$

$$\frac{n' - n}{r} = P^*$$

$$U + P = V$$

When we talked about our simple, thin lenses and their vergence powers, the refractive indices of the media on both sides of the lens were identical — n and n' were equal to 1.0. If, however, the media are not air, we *must* take account of the indices, since they influence the object and image vergences; U does *not* equal $\frac{1}{u}$; it equals $\frac{n}{u}$, and V does not equal $\frac{1}{v}$, it equals $\frac{n'}{v}$.

Originally we declared that the power of a lens P was a certain number of diopters; we should now see that the *total* lens power of such a lens depends on its *two* surface powers, each in turn is dependent on its own radius of curvature and a *difference* in index of refraction. Suffice it to say that with *thin* lenses, the powers of each of the two surfaces simply add together algebraically. A + 5 D front surface and a + 3 D back surface sum to a + 8 D of total power. Similarly, a + 7 D front surface and a − 1 D rear surface would yield + 6 D of total power. (This latter lens is like most ophthalmic corrective lenses in that it is meniscus-shaped, that is, it has a concave surface facing the eye.) So, in general, $P_{total} = P_1 + P_2$. This is for thin lenses. With thicker lenses, however, there is a third, *thickness* factor $[\frac{d}{n}(P_1)(P_2)]$ which must be *subtracted* from the sum of the

* NOTE: To maintain our light convention, r is considered *plus* if the light (moving from left to right) strikes a *convex* surface and minus if it hits a concave surface.

surface powers to arrive at the true "equivalent" lens power. You'll *not* be dealing with thick lenses in this course.

Recall I said you had to know only a *few basic formulas?* Well, here's another of them:

$\frac{n' - n}{r}$ is the refractive power of *any* surface. Fix it in your memory. It tells you that when r is small, the refractive power of the curved surface is high; if r is large, then the power is low; if it is infinitely large, the refractive power is zero — a flat plane is just such a surface, with no vergence power.

PROBLEM:

a) What is the refractive power in air of a polished convex surface of a glass rod ($n = 1.5$) of radius 10 cm?

b) What happens to the power of that same glass rod after immersing it in water ($n = 1.33$)?

ANSWER:

a) $P = \frac{n' - n}{r} = \frac{1.50 - 1.00}{+ .10} = \frac{.50}{.10}$

$P = + 5$ Diopters

b) $P = \frac{1.5 - 1.33}{.10} = \frac{.17}{.10}$

$P = + 1.7$ diopters; thus, the refracting power of the rod has decreased by placing it into water.

When we spoke of thin *lenses*, $P = \frac{1}{f} = \frac{1}{f'}$;

Single surfaces also have focal lengths, and we can use the same reciprocal relationship to power to determine those lengths as we did above, but we must also take into account the refractive indices of the object and image spaces.

$$P_{surface} = \frac{n}{f} = \frac{n'}{f'}$$

This means that if an air-to-water interface has + 5 Diopters of surface power, the f in air is $\frac{1}{+5}$ = 0.20 meters or + 20 cm; however, the f' in the water (the position of sharp focus of an object located

at infinity) $= \frac{1.33}{+5}$ = 26.5 cm to the right of the surface.

This shows that, if the medium on one side of a surface is air, the secondary focal length in the denser medium is n' *times* the primary focal length: $f' = n' f$; this will prove to be a useful relationship.

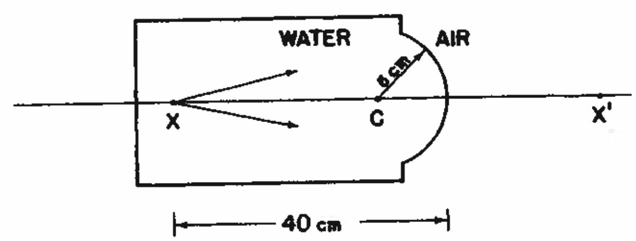
Summary: In any calculations dealing with lens and surface refraction, do not forget that any object and image vergences depend not only on the actual *distances* from the reference plane, but also on the refractive indices of the object and image spaces. We will further explore these concepts when we study the model eye.

PROBLEM:

An object point X is located *in* a water tank and is 40 cm away from the end of that tank, which has a *concave* surface of 5 cm radius. Air surrounds the tank.

a) Where is the image point formed?

b) How long are the primary and secondary focal lengths of the refracting surface?



ANSWER:

a) $U + P = V$

U: The rays are diverging from X as they strike the curved surface, therefore, the vergence is minus at the surface.

The index or refraction of the object space is that of water or 1.33. Thus, U, the object vergence $= -\frac{n_{water}}{u} = -\frac{1.33}{.40 \text{ meters}} =$ -3.33 D.

P: The difference in index of refraction referred to in $\frac{n' - n}{r}$ is always the index of the *image space* minus the index of the *object space:* Here, $(n_{air} - n_{water}) = 1.0 - 1.33 = -0.33$. $r = 5$ cm $= (.05$ m) but is *concave* to the light traveling from left to right, and therefore *minus.* $r = -.05$ meters.

So, $\frac{n' - n}{r} = \frac{-0.33}{-0.05} = +6.6$ D.

(Thus we see that a *concave* surface has *plus* refractive power when the index of the image space is less than the index of the object space.)

V: The vergence of the image rays $= \frac{n'}{v}$; *n' here* is that of the *image space* which is *air:* thus, $n' = 1.0$

$V = \frac{1}{v}$.

Now, solve equation for V:

$$U + P = V$$

$$-\frac{n_{water}}{u} + \frac{(n_{air} - n_{water})}{r} = \frac{n_{air}}{v}$$

$$-\frac{1.33}{.40} + \frac{1 - 1.33}{-.05} = \frac{1}{v}$$

$$-3.33 + 6.60 = \frac{1}{v}$$

$$+3.27 = \frac{1}{v}$$

$$v = +30.5 \text{ cm}$$

There will be a sharp image of X at X', which is located 30.5 cm

to the right of the refractive surface (which is the water tank side).

b) The primary focal length $= \frac{n_{water}}{P} = \frac{1.33}{+6.6}$

$f = +20$ cm in water.

The secondary focal length $= \frac{n_{air}}{P} = \frac{1.00}{+6.6}$

$f' = 15.1$ cm in air.

(Both are measured from the axial vertex of the spherical surface.)

(See Appendix A for a discussion of the influence of the refractive indices of the object and image spaces on the linear magnification.)

ASTIGMATIC REFRACTION

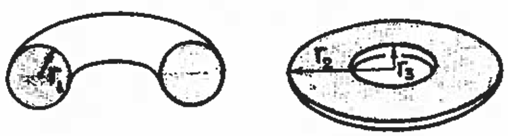
Compared to other icebergs in the sea of optics, astigmatism always seems to offer the student a bit more of a challenge — an obstacle to otherwise smooth sailing. The teaching technique which will serve us as radar to neutralize this impediment has in the past proved to be a most useful one; it allows a firm grasp of the subject *now* while encouraging easy recall *later.* (In my opinion, it is probably Paul Boeder's greatest contribution to optics pedagogy). It presents one particular way of looking at the relationship of the refractive power of a cylindrical lens to the *orientation* of the astigmatic line images formed. But that's getting ahead of our story; we will come back to this shortly.

In our elucidation of $U + P = V$, we learned that P was the power of a spherical surface, that is, one with *one* radius of curvature. If we neglect the lens aberrations of 3rd order optics, that spherical surface can form a point image for each object point. In this type of point-for-point correspondence, the image is said to be stigmatic (point-like). However, not all surfaces are spherical and those that are not, do not usually form stigmatic images. One such surface is called *toroidal* — a particular type of *non*spherical surface which forms non-stigmatic images, but does so in a certain way. It is only for *those* images that we reserve the term astigmatic (*not* point-like).

A toroidal surface is exemplified by the surface of a doughnut. It has *two* fixed radii in contrast to only one for the spherical surface and can be completely described by those two radii. Let us diagram a complete doughnut (torus):

A. WHOLE

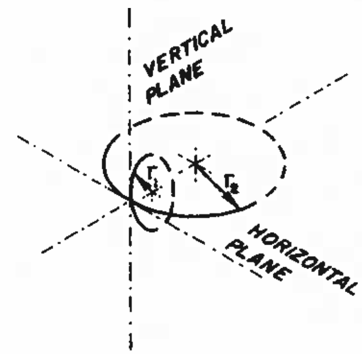
B. VERTICAL SECTION C. HORIZONTAL SECTION



If we cut it in half vertically, we will see two small circles along the cut face (figure B above). Each circle can be described by a radius, r_1.

Compare this, instead, with the torus sliced in half horizontally (figure C above). The cut face now also shows two circles, an outside one of radius r_2 and a smaller, inside one of lesser radius r_3; we will ignore the latter one. All this about doughnuts is nice if you're hungry, but what we're interested in is a description of its outside surface. It is the shape of that front surface that is called toroidal. It is this specific surface which, when treated as a refracting surface (by definition), forms *astigmatic* images.

We can describe the outside surface of the torus as one which is created by the rotation of two radii of unequal length, each in a plane at right angles to the other.



As shown in the figure, r_1 lies in the vertical plane while r_2 is in the horizontal plane. r_1 and r_2 do not have the same center. Their respective centers, however, do lie along a line called the axis of rotation (which is *not* the cylinder axis).

Let us look at each of the two planes drawn in the figure. The curve representing r_1 is a smooth circular outline present in the vertical plane. r_2 and its corresponding circular section are located in the horizontal plane. Each of these should be considered separately, completely independent of one another.

Look at the curve generated only by r_1 in the vertical plane. If it happens to separate two media of different refractive indices, we can calculate its surface power. So, in the vertical plane,

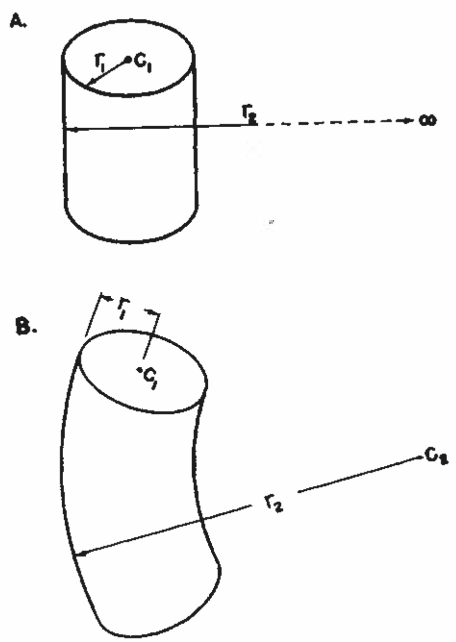
$$P_1 = \frac{n' - n}{r_1}$$

and for any object vergence U we can find the corresponding image vergence V.

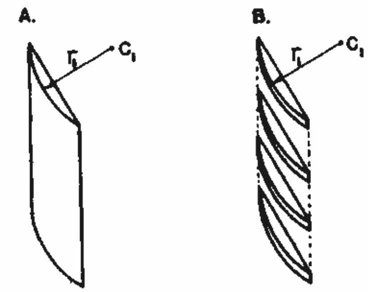
$$(U + P_1 = V_1).$$

We can do the same for r_2 in the horizontal plane, and find P_2, and calculate V_2. So, any object vergence U which impinges on the toroidal surface generated by r_1 and r_2 will be influenced differently by P_1 and P_2, each of which act in separate planes which are at right angles to each other. We are forced to the conclusion that for any given object vergence, a toroidal surface must create two separate images. It is this composite image which is called *astigmatic*.

We can locate the position of certain parts of the image formed by our toric surface by using $U + P = V$ as just shown. But what does this peculiar astigmatic image look like? To find this out, let us investigate the optical effect of a toric surface *other* than that of the *doubly*-curved doughnut; let us use a straight vertical cylinder (as that shown in the accompanying diagram A below), which also happens to be a toric surface, but one with *one* of the radii, r_2, as *infinitely* long. (If r_2 were slightly *shorter* than "infinitely long", you should see that we would have a cylinder which was part of a very large doughnut with a tremendous hole [Figure B].)



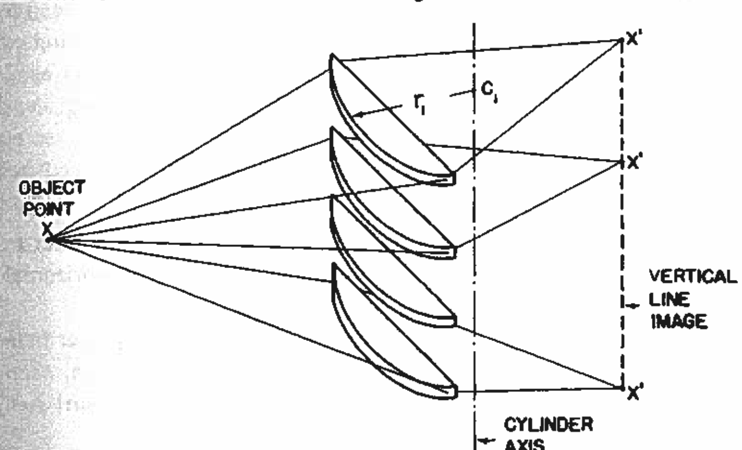
In any case, slice off a thin vertical piece of the cylinder (shown in A above) and it will look like that shown in diagram A below. Then slice the vertical slab into horizontal segments; these will look like a stack of thin, plus lenses (figure B).



Each one of these thin, plus lenses will have the same refracting power, P_1*, say, + 5 D. If we put an object point 50 cm in front of this stack, *each* thin single "lens" would form a point image (see figure below). Where is this point image (X') located?

$$U + P_1 = V$$
$$-2 + 5 = +3$$

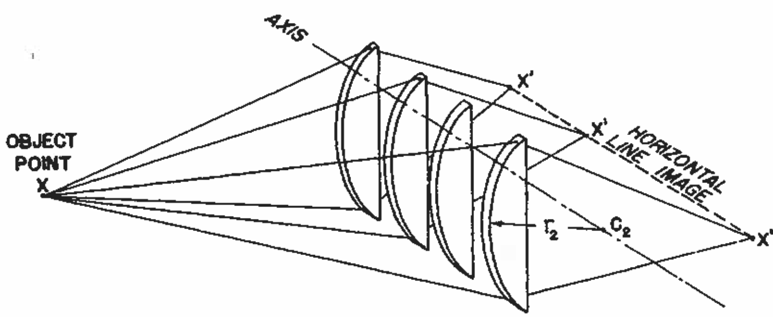
So, each point image X' will be 33 cm away. Since every one of our slices will also form its own point image of X, the final composite image of X will have to be a series of X' positions oriented in a straight vertical line. We say that our complete right angled cylindrical lens forms a real vertical line image at X'.



* P_1 refers to the power in the horizontal plane (where r_1 is located) as shown in the diagrams above.

Note the vertical line drawn through the center where r_1 originates. *That* dotted line is called the *cylinder axis.* The distance from that line to the cylinder surface *is* r_1 and, of course, is one of the variables which gives refracting *power* to that surface. This power of P_1 diopters lies only in the *horizontal* meridian as shown. The axis, however, is *vertical.* In ophthalmologic jargon, we say that P_1 is the power *associated with* the vertical axis (at 90°), or, in shorthand, $P_1 \times 90°$. So, tying these concepts together, we have learned that the cylinder power $P_1 \times 90$ creates an image which is a *vertical* line. (Keep the association *"vertical line image* and *axis 90°"* sharply in mind.) Such a vertical image line is produced for each and every object point.

Let's look at another right cylinder but of a different, longer *r*. Again, shave off one face and turn it so that its axis (the locus of all the *r_2 centers*) runs horizontally. Further, slice this piece vertically so that our slab is now composed of a series of plus lens segments stacked side by side (as shown).



When point X is imaged, *each* of these lens segments will act to form a *point* image X'; when all are considered together, a horizontal line image is composed.

Since we chose r_2 to be longer than r_1, P_2 will be of *less* power than P_1 in the last example; let's arbitrarily say P_2 is + 3 D, that is, each vertical slice (representing P_2 in the vertical — 90° meridian) has plus lens power of + 3 D.

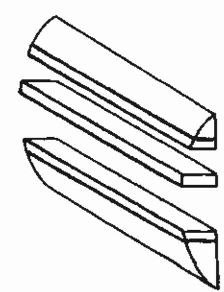
With object point X located 50 cm from this cylindrical lens, $U = -2$ D.

$$U + P_2 = V$$
$$-2 + 3 = +1$$

Thus, the location of the horizontal line image is 1 meter away from the lens. And again we see that power P_2, though it *acts* in the vertical meridian, is associated with a *horizontal* axis (that is, $P_2 \times 180°$*)* and so, focuses a *horizontal* line image.

In *both* the right cylindrical lenses considered so far (the $+3 \times 180$ and the $+5 \times 90$ we looked at initially), we dealt with the power in only one of the meridians — that exerting + 3 D and + 5 D, respectively. Any and every cylindrical lens will, however, always have *two* "major" meridians — one with maximal spherical power and another (at right angles to it) of minimal power. Each of these major meridians will form a line image, so, there will always be two line images formed by any cylindrical lens. In our examples above, we showed you the location of only one of them. Where is the other hidden? To find out, let's examine our second lens ($+3 \times 180$), shown in the last diagram, and specifically look at the meridian which is responsible for imaging the *vertical* line — that is, the horizontal meridian.

We had sliced this lens into vertical segments to study the vertical meridian, so now let's cut the slices horizontally to examine the horizontal meridian, (see figure below). Each one of the horizontal slices of this segment would yield a piece of material with parallel faces having no curvature and, therefore, no refractive power; thus, we can conclude that the horizontal meridian (180° meridian), which also happens to be the direction of our cylindrical lens *axis,* has no power.



Now, locate the vertical image line with the object point still kept at 50 cm.

$$U + P = V$$
$$-2 + 0 = -2$$

Thus, the vertical line image, which we know must be present with this cylindrical lens, is optically located *at* the same position as the object point (but, in the image space).

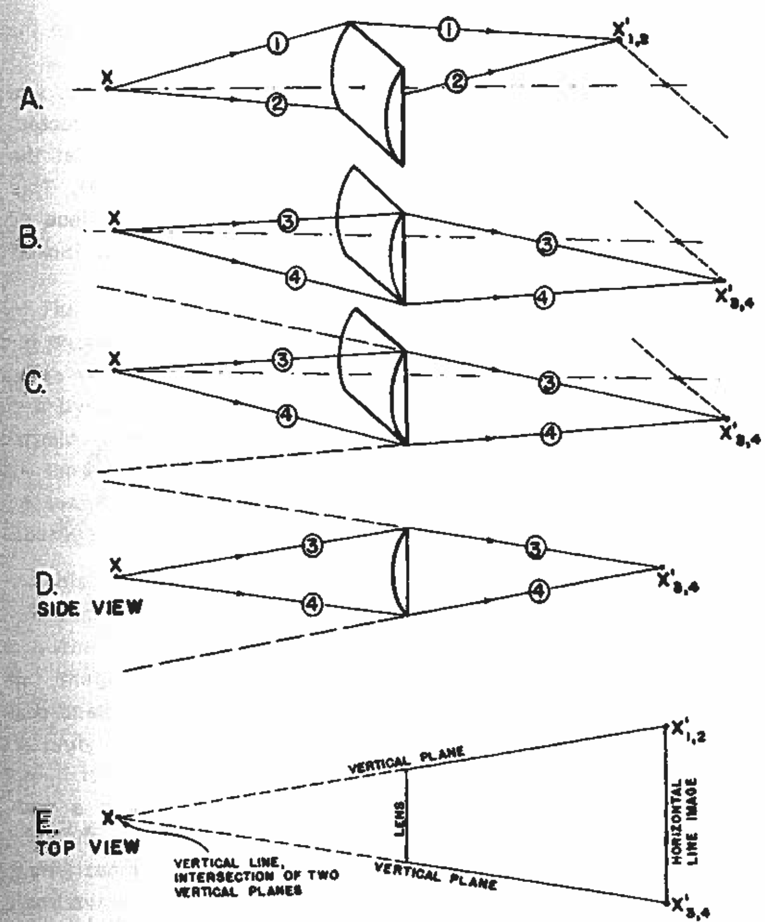
The Maddox Rod

This somewhat surprising finding forms the basis for one of the most useful of optical tools — one which you will probably use every day — the Maddox Rod. So, pay attention and please follow through the next few optical diagrams. These will enable you to understand how this instrument works. We will use the same right cylinder $P_2 \times 180$ as above and diagram the object and image rays originating from object point X.

In the series of diagrams, object point X lies on the optical (*not* cylindrical) axis of this lens. Select 4 rays emitted by point X towards the 4 corners of our right cylinder whose axis is horizontal — first, rays 1 and 2 (diagram A). These rays lie in a vertical plane on the far side of the lens and are converged to $X'_{1,2}$ by the lens. In the image space, these rays will remain in the same vertical plane as they were originally.

In diagram B, rays 3 and 4 lie in another vertical plane on the *near* side of the lens and are also converged, but to a point $X'_{3,4}$. We have already seen that the *composite image* of X is a horizontal line. $X'_{1,2}$ and $X'_{3,4}$ are the point extremes of this horizontal line image.

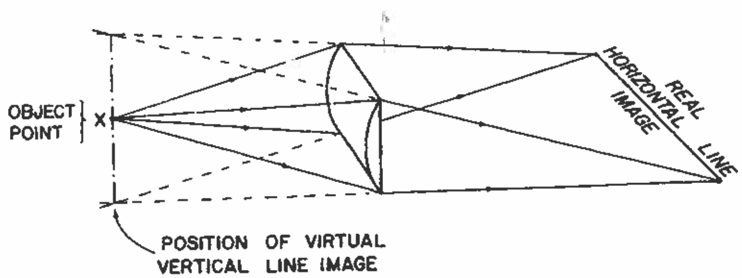
All converging rays (in the image space) can be extended backwards. Let us do so from $X'_{3,4}$ (diagram C). These extended rays (as shown) will form a triangular shaped, vertical plane with the apex at point $X'_{3,4}$ and bounded by *image* rays 3 and 4. These rays will lie in the same plane as the *object* rays 3 and 4, since objects and images always lie in the same plane. Object point X will, of course, also lie in this plane. (Diagram D shows a *side* view of this vertical plane, with pertinent positions labelled).



Follow this same procedure for object and image rays 1, 2 and point $X'_{1,2}$. This will create a different vertical plane, but also will include object point X. These two vertical, triangular planes will intersect in a vertical line located at the same position of object point X.

You can see this somewhat more easily if you look down on the diagram from the top (diagram E).

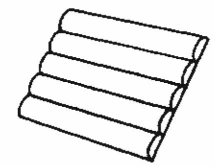
If we now draw the full optical diagram (see next figure), you should be able to understand how each of the rays is projected. Note the virtual vertical line image, which must be located at the same position as object point X (but in the *image* space). This vertical line image is virtual and cannot be focused on a piece of paper; however, it can be photographed or seen, but only when looked at *through* the cylindrical lens towards object X.



This rather long preamble is necessary for you to understand the Maddox Rod: this instrument is simply a high-powered plus cylinder. If held in front of an eye which is looking at a point source of light, and if the cylinder axis is horizontal, the lens will create a real *horizontal* line image close behind the lens. However, since the eye is *very* close to the lens, the horizontal line cannot be seen; (it is too close to be focused upon). The eye then will see the virtual, *vertical* image line apparently located at the object point. (Usually, the object light is distant, but it may also be nearby, as it was in the previous diagrams — in either case, the vertical line image will always be located wherever the object light is).

In practice, instead of using a single, high powered cylindrical lens, a *series* of high powered cylinders are stacked side by side with their axes aligned as shown below. This will make the vertical line image appear longer, and therefore, more visible to a patient.

MADDOX ROD



The Maddox Rod is used to measure extraocular muscle balances and phorias. When it is presented to one eye, the rod makes a *line* image visible to that retina while the point light source itself is seen by the other; this "breaks fusion" between the two eyes and permits them to settle into their tonic (relaxed) positions. Thereby, any tendency to ocular misalignment can be detected and, if present, measured. This is done by determining (in prism diopters) the angular separation which appears to exist between the two dissimilar images.

This detailed elaboration of the imaging of an object point by a cylindrical lens should have made the following quite clear: even a lens which has no power in one of its primary meridians will form *two* line images and each line image will be parallel to one of those primary meridians.

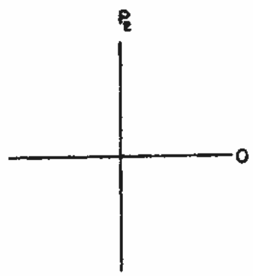
The Astigmatic Cross Diagram

So far we've dealt with the real image line formed by a simple cylindrical lens. We paid more attention to the axis than the power since it was the *axis* that had the same orientation as the image line. However, the lens *power* belonging to that axis is what actually places that line at some definite location within the image space. So, let's now discuss not the image line but the cylindrical lens itself.

When I talk about a cylindrical lens, say $P_1 \times 180$, I mean one which has a *maximum* refractive power of P_1; though it functions by putting the *horizontal* image line into some position, that *power* is located in (and acts in) the vertical meridian, that is, at 90°. Note that I said "meridian". I can talk about a certain meridian because

I wish to identify one specific *direction.* This direction does not automatically associate with any *particular* lens power; it *could* be zero if it happens to represent the location of the axis, or it might represent any other sectional power of that lens. In *this* lens ($P_2 \times 180$), the vertical meridian (*at* 90°) represents the plane which contains power P_2.

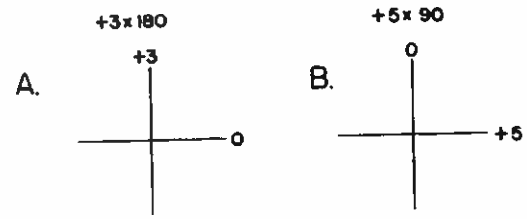
This same information can be written as "P_2 acts in the vertical meridian", or "P_2 is located in the vertical meridian" or more simply "P_2 @ 90°" — all these are equivalent expressions. Furthermore, I can draw a "cross" diagram (which portrays the two major lens meridians) and label P_2 along the vertical meridian (see below).



This lens' *axis* (× 180°) is located in the horizontal plane of the lens and is shown in the diagram as being present along the horizontal meridian with "zero refractive power".

Thus, any lens can be shown on a "cross", with two major meridians labelled. A *spherical* lens would show both of these meridians labelled with identical powers. Any "cross" which shows "zero power" along one of its major meridians must clearly be a simple *cylindrical* lens and that meridian must be the *axis.*

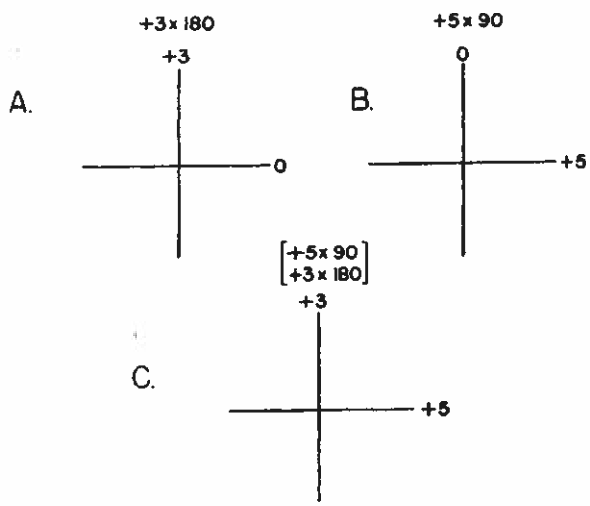
Let us now use the "cross" diagram to represent the *powers* in each of the primary meridians of the cylindrical lens we have already discussed, ($P_2 \times 180$). (See figure A below).



Since the vertical meridian has + 3 D of power, it is labelled so, and the horizontal, 0 D. We should, just by looking at the "cross", be easily able to write the formula of this simple cylinder as + 3 × 180°.

If you will recall, our *first* cylinder lens example was oriented with the axis situated vertically ($P_1 \times 90$). This lens, of formula + 5 × 90°, would be drawn on the "cross" in the manner shown in B above. Each of these lenses is a simple right cylinder with no refractive power given by the axis meridian. We have already considered how each lens imaged a point object 50 cm away. To reiterate, the + 5 × 90° cylindrical lens imaged the point as a vertical line 33 cm from the lens. The + 3 × 180 lens created a horizontal line image 1 meter away. In addition, we should now know that when each of these lenses is considered separately, it also forms another, but *virtual* line image which is located at the same position as the original object point and oriented perpendicular to the cylinder axis.

Without too much difficulty, we should now understand what happens if we put both cylindrical surfaces *simultaneously* onto *one* piece of glass. Instead of a right cylinder with a straight vertical face (no curvature), we would now have one which has *some* curvature vertically, as well as horizontally. We can still slice the lens into horizontal segments and obtain pieces with power P_1 just as we did with the right cylinder. When, however, we slice this glass vertically, the segments will now *have* power P_2. Such a combination lens will act like our two separate right cylindrical lenses — the + 5 × 90 and the + 3 × 180 — in forming their line images as already discussed, but this lens will do so simultaneously. (However, there will be no virtual image lines at the object position as with the Maddox Rod.)



RESULTANT: COMBINED CYLINDER FORM

The diagram of these two separate cylindrical lenses (shown in A and B above) can be combined on a resultant "cross" (C above) to yield a single lens in which the power acting in each of the major meridians is simply added algebraically. This resultant represents the cylindrical combination which is written $+5 \times 90$ *combined with* $+3 \times 180$, or $\left[\begin{smallmatrix} +5 \times 90 \\ +3 \times 180 \end{smallmatrix} \right]$.

Conversely, any "combined" cylindrical lens can always be re-divided into two simple cylindrical lenses, as those shown above. (You will get more experience with this later). But remember, each cylindrical lens will be responsible for creating a focal *line* for every object point, and each of the lines will have the same orientation as

the *axis* associated with the power which imaged it. This is *the* most important practical point about cylindrical lens imagery, and you must know it backward, forwards and should be able to recite it even if you are awakened from a sound sleep; once again, it is the meridian of the *axis* that determines the orientation of the particular line-image created: So,

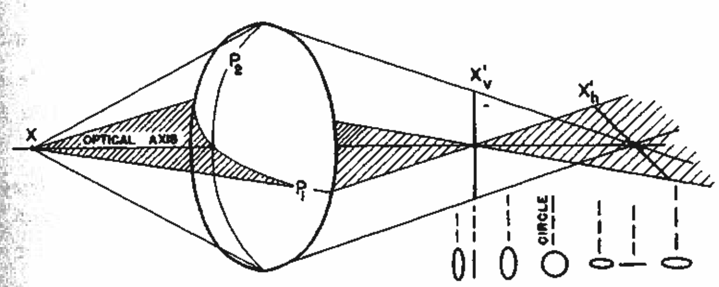
$P_1 \times 90$ forms a *vertical* line image for each object point

$P_2 \times 180$ forms a *horizontal* line image for each object point.

The Circle of Least Confusion

Again back to our example with the object located 50 cm away, the $+5 \times 90$ forms the vertical line image 33 cm from the lens and the $+3 \times 180$ forms the horizontal line image 1 meter away. What about the character of the image located in between the two focal lines?

The following diagram should help to answer this question:



APPEARANCE OF THE IMAGE OF POINT X ON A SCREEN PARALLEL TO THE LENS

We are showing the same "combination of cylinders" lens we have been talking about, but here it is shaped more like a conventional "lens".

$$P_1 \times 90 = +5 \times 90$$
$$P_2 \times 180 = +3 \times 180$$

From object point X, only four rays are drawn to keep the diagram simple: two are in the horizontal plane (which is lightly shaded). These rays proceed to the horizontal edges of the lens. The other two rays shown are in the vertical plane. P_1 (with its *axis* at 90°) *acts* in the horizontal plane but focuses the vertical line at X_v' as shown. (Remember, × *90* means we will be dealing with a *vertical* line image). Similarly × *180* means we will have horizontal line imagery with power P_2 placing the horizontal line into position.

In the figure above, we are given a bright object point and an image by the lens. If we place a screen parallel to the lens plane in the image space at position X_v', we will see a sharp, bright vertical line there (surrounded by some dim, diffuse light); if the screen is moved to position X_h', we will have a horizontal line. In between these two line images, other shapes are present; we will find bright spots of light shaped as shown in the diagram — vertically oval when in the proximity of the vertical line and horizontally oval when near the horizontal line image. At one discrete screen position between the two line images, we will find a perfectly circular spot; remember that this spot of light represents the image at that particular screen position of only *one* object point. Since the spot is here circular, it more closely represents a desired "point" image than does either *line* image or some *oval* image. This image is called "the circle of least confusion" and represents the axial position which provides an image which is less "blurry" overall than any other.

The exact position of the "circle of least confusion" is *always* one-half way *dioptrically* between the two line foci (that is, with a vergence which is the *average* of X_v' and X_h'). Clinically this is a most important vergence.

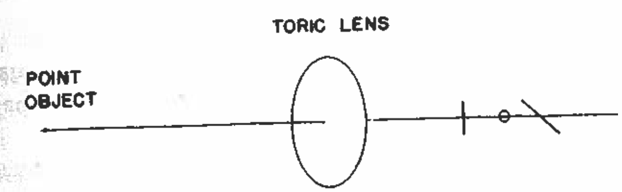
In *linear* distance the "circle" is always somewhat closer to that image line which is nearer the lens; for example, with the object at 50 cm, X_v' had an image vergence of + 3 D and X_h' an image vergence of + 1 D. The "circle of least confusion" has a vergence of $\frac{1}{2}(+3+1) = \frac{+4}{2} = +2$ D and, thus, is located 50 cm from the lens. But, X_v' was 33 cm away and X_h' was 100 cm away; so you can see that the "circle" image is closer to (only 17 cm from) the vertical line, and 50 cm from the more distant, horizontal line. It will

always be closer to that image line which is most proximal to the lens. But, *vergence-wise*, the "circle of least confusion" is always halfway between, and the lens power which is responsible for imaging this circle is called the "spherical equivalent" power; that power is one half way dioptrically between the maximum and minimum meridional lens powers. *That* is the item you must remember. We will encounter the spherical equivalent later in reference to "cross-cylinders".

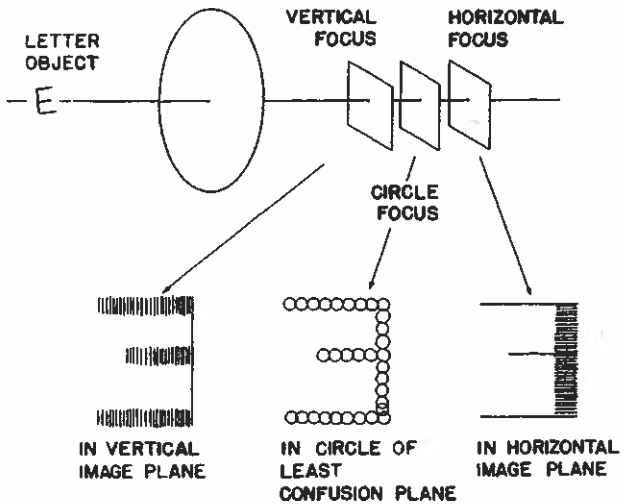
The entire image which stretches between the image lines (including the "circle") is known as *Sturm's interval* and is often referred to as such. (J.F.C. Sturm investigated astigmatism in 1838).

Images of Extended Objects

So far, we have spoken only of the representation of object *points* in the astigmatic image. What about the images of *full* objects? Let us schematize first again; for a *point* object:



Now, instead of considering only one object point X, as above, let's put a letter E at that same position.



Every point making up the E will form a vertical image line in the vertical image plane, where the image of the E looks like that shown on the left. (Notice that it's upside down, as are all real images of an upright E). The vertical, right-hand edge is sharp because all the individual vertical line images fall superimposed on each other.

In the horizontal image plane (shown on the right), every object point in the E will give rise to a horizontal image line. Again, the horizontal bars on the E are sharp since each horizontal image line falls on top of the adjacent one, reinforcing it.

In the plane of the circle of least confusion (center above), the images of *each* object point are *circles*; this image more closely represents the true shape of the actual object E than do either of the other two images.

If the astigmatism were such that the two cylindrical axes were not oriented vertically and horizontally but were at some other positions, each image line (for every point in the object) would still be parallel to the corresponding axes — let's say 45° and 135°. The

images might look somewhat like those below; notice that neither the vertical nor horizontal parts of the E are clear, but, the image in the plane of the circle of least confusion would, of course, look exactly like that shown above.

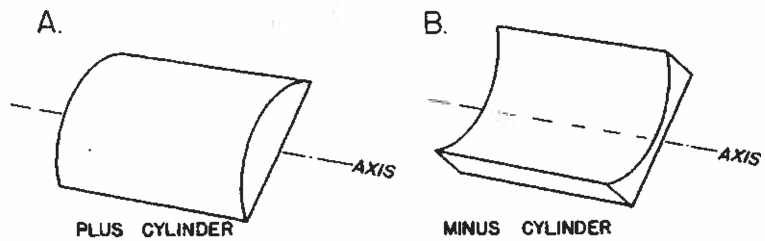
At this point we will skip learning what actually determines the *length* of the image lines and *size* of the "circles of least confusion". This will be taken up later in a discussion of the use of the stenopeic slit in clinical refraction.

Differential Motion of Image Lines

Without any difficulty whatsoever, we should now be able to find the locations and orientations of the image lines. This is particularly easy to do when we express the cylindrical lens as a combination of two cylinders, $(P_1 \times 90 \supset P_2 \times 180)$. So, let's place a new cylindrical lens in front of the above combination. This new lens will add directly to the combination lens in a particular way. Assume that its axis is *parallel* to either one of the others; (if it is not, the cylindrical addition becomes more complicated — we will broach this latter subject later on.) For the moment, then, we will hold our new cylindrical lens P_3 so that its axis is at 180°. Now $P_3 \times 180$ will add only to that power already present which has its axis also at 180°; that is, $P_3 \times 180$ adds only to $P_2 \times 180$ and yields

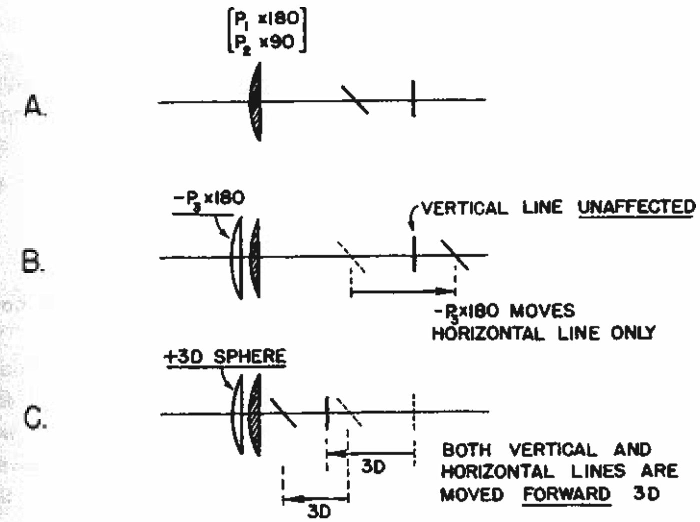
$(P_2 + P_3) \times 180$; the $P_1 \times 90$ is *not affected at all.* Since $P_2 \times 180$ was the power influencing the position of the horizontal line image, the new $P_3 \times 180$ will move *only* that same horizontal line.

P_3 can be either plus or minus. We are already acquainted with a *plus* cylinder × 180 (A below); A $-P_3 \times 180$ lens is called a *minus* cylinder × 180 and would look like diagram B.



Either of these two cylindrical lenses would move only the pre-existing horizontal line, but each would move it in the opposite direction: the *plus* cylinder would "pull" that horizontal line image closer to the lens itself since it adds plus vergence to the image line; the *minus* cylinder will decrease the image vergence of that line and "push" it further away. (Notice, we *cannot* automatically say that either lens will move the horizontal line closer to or further from a *vertical* image since that depends on where that vertical line is situated!)

Let's look at a problem: If I show you a cylindrical lens combination where the image-lines are at the positions shown in figure A below, you should quickly be able to tell that the power associated with axis 180° ($P_1 \times 180$) must be more *plus* than power P_2 *with* its axis at 90°. You know this because the horizontal image line lies closer to the lens than the vertical one.



If I place a $-P_3 \times 180$ in front of (and in contact with) this lens (figure B above), I will move the horizontal line back away from its present position and towards the vertical line. (Figure B shows the horizontal line being moved beyond the position of the vertical). You can see that if $-P_3$ is of just the "proper" strength in minus power, it will move the horizontal line exactly to the location of the vertical. This would completely "collapse" the conoid of Sturm, and thus eliminate all the astigmatism present. At that moment, the sum of P_1 and P_3 (both with their axes at 180°) will equal P_2 with its axis at 90°; that is, the *total* power in both the 90° and 180° meridians will be equal. It should be obvious that this total combination will be equivalent to a *spherical lens.* (A + 7.50 D spherical lens can also be written as +7.50 × 90 ⊂ + 7.50 × 180).

So we have a beautiful mechanism for moving whichever image line we wish. Use simple cylindrical lenses oriented with the axis parallel (aligned in the same direction) as the image line you want to move. (Remember, plus cylinders move the line closer while minus cylinders push it away). The *other* line will stay put!

If instead of adding a simple cylinder ($P_3 \times 180$) we add another "combined" cylindrical lens ($P_4 \times 90 \supset P_5 \times 180$), all we have to do is add the powers associated with identical axes to obtain the correct *resultant* lens power:

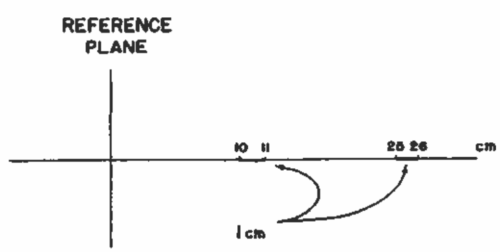
When lens $P_1 \times 180 \supset P_2 \times 90$
is added to $P_5 \times 180 \supset P_4 \times 90$
you obtain $(P_1 + P_5) \times 180 \supset (P_2 + P_4) \times 90$.

The combined powers associated with each axis will together move their corresponding lines.

Instead of adding *cylindrical* power to a "combination" cylinder lens, let's add only spherical power. Since the spherical lens will add equal powers to both meridians simultaneously, both image lines will be moved *equally*, but only in a *dioptric* sense. (See figure C above). Thus, if $+3$ D sphere is added to $+3 \times 180 \supset +2 \times 90$, the resultant would be

$$(+3+3) \times 180 \supset (+2+3) \times 90$$
$$\text{or} \quad +6 \times 180 \supset +5 \times 90.$$

Although both lines would be moved toward the lens an equal *dioptric* distance (each by $+3$ D), the *linear* distance moved by the more distant line must be greater than that moved by the closer line. This simply shows the effect that proximity (to a reference plane) has on vergence, something you learned in the first few pages of this book. Look at the following figure:

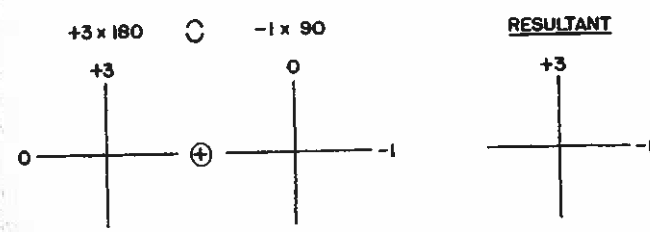


A 1 cm linear distance between 10 and 11 cm corresponds to a dioptric change of $10 - 9.1$ or 0.9 D, while that same 1 cm distance between 25 and 26 cm corresponds to a dioptric change of $4 - 3.85$ or 0.15 D. The addition of a full *diopter* of plus *at the reference plane* will "pull" both sets of points forward but will exert a greater movement effect on the more distant pair of points.

Transposition of Cylinders

We now have sufficient background to understand the different ways we might describe a cylindrical lens combination.

On the "cross" diagram, we showed that a $+3 \times 180 \supset -1 \times 90$ would look like the following with the resultant lens shown below:

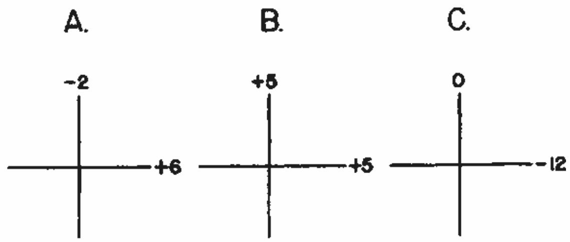


This resultant is *read* as "$+3$ @ 90° (or $+3$ D in the vertical meridian) combined with -1 @ 180°."

The resultant is the *key* way to describe any lens — just note the dioptric power of the two major meridians.

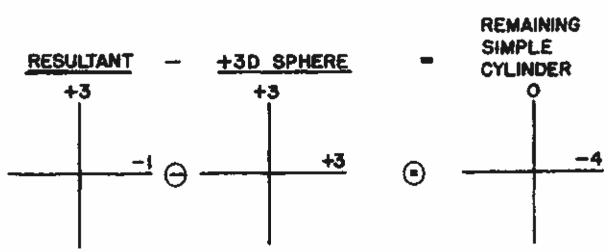
We have expressed the above lens as a combination of two simple cylindrical lenses ($+3 \times 180$ and -1×90). This same resultant can also be divided into other combinations — just as long as the algebraic sum of meridian powers yields the same powers shown in the major meridians of the resultant lens.

If we wished, we *could* subdivide the same resultant shown above into any number of other lenses; for example, the following three:

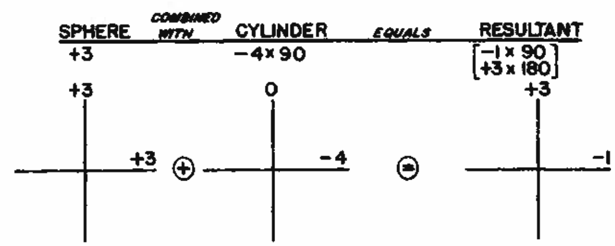


A. − 2 @ 90 + 6 @ 180 B. + 5 @ 90 + 5 @ 180 C. 0 @ 90 − 12 @ 180

Since the algebraic sum in each of the major meridians is $\left[\begin{smallmatrix} +3 @ 90 \\ -1 @ 180 \end{smallmatrix}\right]$, the resultant of this 3-lens combination is still the same as the one given in our last example. However, let's get back to a more practical "factoring" of the resultant. It is conventional to divide the resultant into two lenses, one of which is a sphere, the other a simple cylinder. We can determine the spherical component contained in our resultant by subtracting, from *both* meridians, the power shown at either of them. For example, the two meridional powers here are + 3 D and − 1 D. We can remove + 3 D of sphere from the resultant by subtracting + 3 D from both the vertical and horizontal meridians; we would then have the situation below:



It is clear, then, that our same resultant can be considered to be composed of a + 3 D sphere combined with a − 4 × 90 simple cylinder. (To check, just add the two major meridians algebraically; you should obtain the original resultant's major meridional powers).



Written longhand, these same two lenses can be stated in a *spherocylindrical* form:

+ 3 D sphere combined with − 4 D cylinder axis 90°

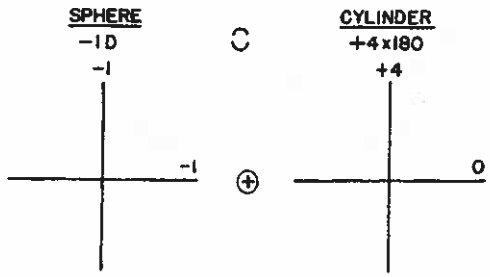
or better, + 3 ⊃ − 4 × 90°;

or even shorter, + 3 − 4 × 90°.

This is a spherocylindrical shorthand for precisely the same resultant lens which also represented our original "combination" of two cylinders. This form is called the *minus* cylinder form since the cylinder sign is *minus*. It is obtained from the resultant by choosing to extract a sphere of such power so as to leave one meridian remaining with minus power.

Return again to our resultant; we could, instead of extracting a + 3 D sphere, have separated out a − 1 D sphere — representing the other meridional power.

This would give us the two following lenses:



(Again, a check by adding each of the corresponding meridian powers would yield the same powers evident on the resultant).

These lenses also can be written in a spherocylindrical form:

— 1 D sphere combined with + 4 D cylinder axis 180°,

or $-1 + 4 \times 180°$.

This is called the *plus* cylinder form.

Both plus and minus cylinder forms are convenient ways to write down the corrective lens prescription for a spectacle lens. Both expressions represent single lenses that would focus light identically on an optical bench or for a patient's eye. However, only the *powers* of the major meridians are explicit in these prescriptions. The actual *shape* or surface curvatures of such a combination lens is *not* denoted by these formulas; many *different* shapes or forms of proper prescription lenses *could* provide exactly the same refractive powers for each meridian. So, don't think you are prescribing a specifically shaped lens with your notation. You are only indicating specific powers, which will be identical for any of the ways in which you can write the prescription.

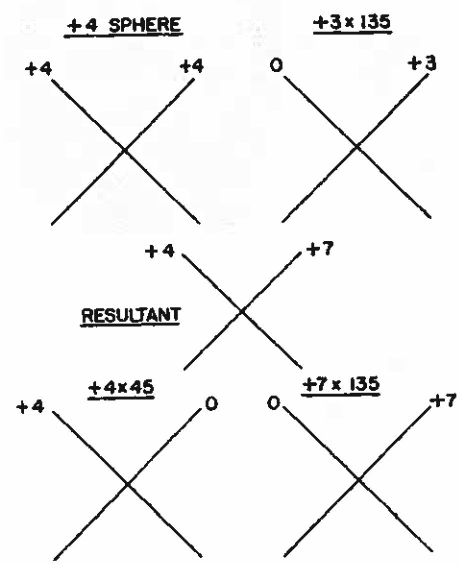
The plus and minus cylinder forms can easily be interconverted. If a minus cylinder lens is $+7 - 3 \times 45$, it can be changed to *plus* cylinder form; to obtain the new sphere, algebraically add the *sphere* and *cylindrical* powers — $(+7) + (-3) = +4$; the new sphere will be + 4 D. The cylinder sign is simply changed to the opposite sign (— 3 D becomes + 3 D) and the axis is rotated 90° — thus 45° becomes 135°. The plus cylinder form of this lens is $+4 + 3 \times 135$.

The great advantage of both the plus and minus spherocylindrical forms is the *immediate* accessibility of the amount of cylinder and its axis; however, if you want to get some insight to the positions and directions of the *image lines* (which we *will* when considering the clinical aspects of cylinders), the spherocylindrical form is not as useful as the third form — "the combination of the two cylinders". It is this latter form which gives us more complete flexibility.

To find the third prescription form requires a minimum amount of arithmetic; and this always seems to be a pitfall leading to possible errors.

My own favorite way to change the lens to this form is by way of the "cross". Any errors are much less likely with this simple graphical means than with an algebraic one.

To convert from the spherocylinder $+4 + 3 \times 135$, first jot down both sphere and cylindrical "crosses"; then form the resultant by adding the corresponding meridians and finally, write each meridian power as a separate simple cylinder.

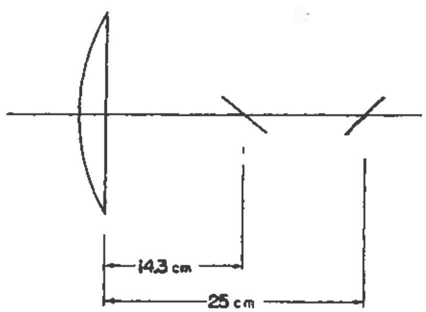


This is the required conversion:

$$\left[\begin{matrix}+4 \times 45 \\ +7 \times 135\end{matrix}\right]$$

Again, you can check by adding the powers in the major meridians; you should arrive at the resultant lens.

Remember we are still talking about *one* lens — we are *describing* it as a combination of two cylindric lenses only to make it easier to visualize its action on light. This lens would take an object point at ∞ ($U = 0$) and form two focal lines; one is located $+\frac{1}{4\,D}$ or 25 cm from the lens and oriented at *45°* (just like the axis); the second line is located $\frac{1}{7\,D}$ or 14.3 cm from the lens and oriented at 135°.

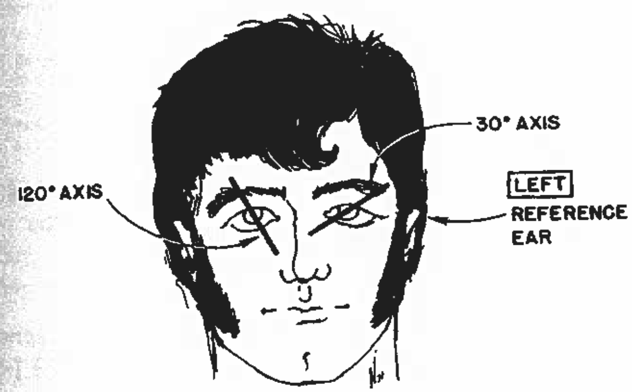


Now, I think the beauty of this way of expressing a spherocylindrical lens should be apparent, since it enables you to visualize the focal lines clearly. However, you should, at the drop of a hat, be capable of transposing cylindrical lenses into *any* of the three forms shown.

Here, we should point out what the usual clinical axis-convention is. You will use it every day when refracting a patient, and you will just have to memorize it. The 0° — 180° is the horizontal meridian.

0° begins at the patient's *left* ear and rotates counterclockwise when you are facing the patient; this holds true for *both eyes.*

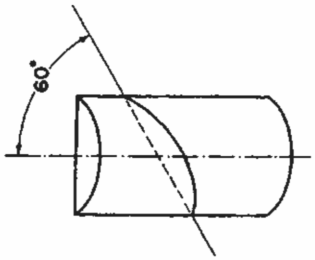
Two sample meridians are shown below. Each designates one particular axis.



Meridional Powers of Cylindrical Lenses

We have just seen that cylindrical lenses vary in their refracting power, going from a maximum dioptric power in one major meridian to a minimum power at a second major meridian which is always perpendicular to the first. But what about the powers in between?

The power gradation going from maximum to minimum meridional power is *not* a straight line change; the power gain moving from the *axis* meridian (which *is* minimum) to the maximum one increases by the *sine*² of the angle away from the axis.



In this 3 D cylinder, the power in the meridian *60°* to the *axis* is

$$3\,(\sin^2 60°) =$$
$$3\,(.866)^2 =$$
$$3\,(.75) = +\,2.25\text{ D}$$

If there is an accompanying *sphere* in the lens, *its* power must be added to that found in the cylinder's meridian to obtain the *total* power exerted by that specific meridian. Try one for yourself:

PROBLEM:

What is the dioptric power in the 45° meridian of the sphero-cylinder combination + 2 − 5 × 90?

ANSWER:

Sphere: + 2.00 D in all meridians

Cylinder: The angle between the cylinder axis and the required meridian is 45°.

$$-\,5\,(\sin^2 45°) =$$
$$-\,5\,(.707)^2 =$$
$$-\,5\,(.5) = -\,2.50\text{ D at }45°$$

Therefore, the *total* dioptric power (at 45°)

$$= +\,2.00 - 2.50 = -0.50\text{ D}$$

For *this* lens there happens to be an easier way, which depends on a fact: For any spherocylindrical lens, the power in the meridian 45° to the axis (that is, halfway between the maximum and minimum meridional powers) is always the spherical equivalent of that lens — half of the cylinder added to the sphere ($\frac{1}{2} \times (-5) + 2 = -0.50$ D). We will study much more about "spherical equivalent" later.

Knowing this sine² relationship will help you determine the dioptric power in *any* meridian of *any* cylindrical lens. This may be useful to

you when dealing with bifocals later, since you must know the total refractive power at a particular point in a lens to be able to calculate the *prism* power induced there.

The table below shows some selected meridional powers (in diopters) of a 1 diopter cylinder, axis 180°. (These power figures denote the sine² of the angle between the specific meridians mentioned and the cylinder's axis.) The meridional powers of cylinders of higher power than 1 D are directly proportional to those shown.

Meridian	*Power*	*Meridian*	*Power*
0° and 180°	.000	50° and 130°	.587
10° and 170°	.030	60° and 120°	.750
20° and 160°	.117	70° and 110°	.884
30° and 150°	.250	80° and 100°	.970
40° and 140°	.413	90°	1.000
45° and 135°	.500		

With this exposure to cylindrical lenses and astigmatic imagery under your belts, you should be well equipped to handle clinical astigmatism and refraction. We will delve deeper, when appropriate, into further aspects of this fascinating field.

REFLECTION

Somehow we're going to have to squeeze in another subject area that seems slightly out of place; yet it is important, so here goes.

When we first started out, we showed how the vergence of light rays emanating from some object was influenced by lenses, each type of lens adding or subtracting its own particular vergence power to that of the incoming light and thus creating an image. There are *other* surfaces which (like lenses) can also change vergence. These accomplish this feat *not* through the *refraction* of rays (via Snell's Law) but through the *reflection* of rays.

Reflected rays can form an image (which also may be real or virtual) in much the same manner as refracted rays, and reflection should be considered as a "special case" of refraction — that is, subject to many of the same rules we have laid down for refraction. The difference is that *reflected* rays are bounced off a *smooth* (flat or curved) surface in an absolutely characteristic manner, with the angle of incidence *i* always equal to the angle of *reflection r* — this is called *regular* or *specular* reflection.