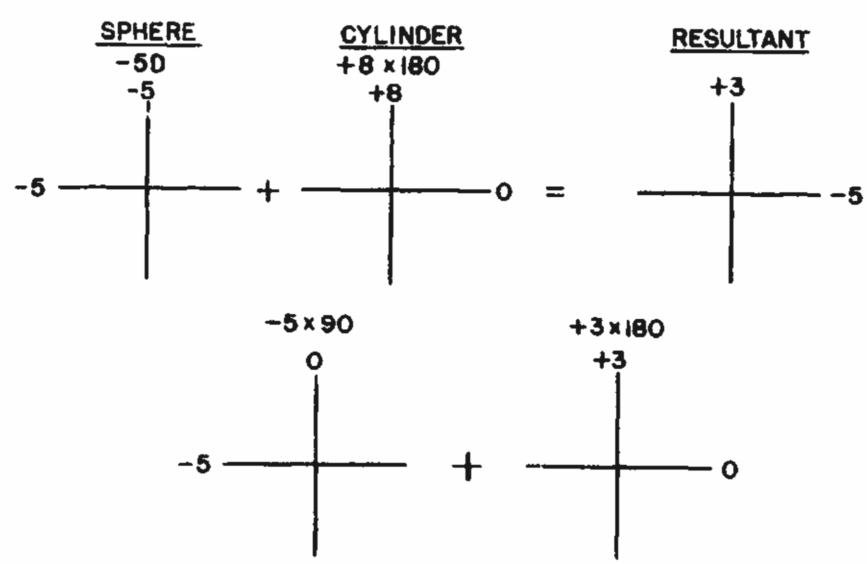
is 8 D. The circle of least confusion, which is dioptrically halfway between these lines, must be located 4 D from each; but, that position is 1 D in *front* of the retina.

So, with the eye *error* expressed in the combined cylinder form, you should never have difficulty visualizing the lines. But, the eye *error* in the form given above is not just lying around like a call-girl waiting for a jingle. It is partly hidden, and requires a little effort — some seduction — on your part. The type of information that *is* usually available is the prescription of the *corrective* lens; (again, consider it to be located at the cornea). — 5 + 8 × 180 would be the typical spherocylindrical form of such a lens. Now, you should *still* be able to sketch and locate the positions of the line foci and the circle of least confusion in this eye with that corrective lens *removed.* Your brain should conjure up the *exact same diagram* as above. How do you get there from here?

As you know, we can express this corrective lens in two other ways — you should be able to derive them now, but I'll "spoon-feed" once more.

Since it is much easier to visualize the positions of the line foci when you deal with the combined cylinder form, transpose the spherocylindrical corrective lens using the "cross" diagram.

A — 5 + 8 × 180 lens placed on the cross is as follows:



The resultant is shown above, as well as the derived two simple cylinders. Thus, the transposed form for this *corrective* lens is

$$\begin{bmatrix} -5 \times 90 \\ +3 \times 180 \end{bmatrix}.$$

But, a -5×90 *corrects* ("neutralizes") an eye *error* of $+5 \times 90$, and a $+3 \times 180$ cylinder corrects an error of -3×180. So, the composite eye *error* requiring this corrective lens is

$$\begin{bmatrix} +5 \times 90 \\ -3 \times 180 \end{bmatrix}.$$

We have already seen in our last diagram that this *error* produces the same line foci in the exact positions shown there.

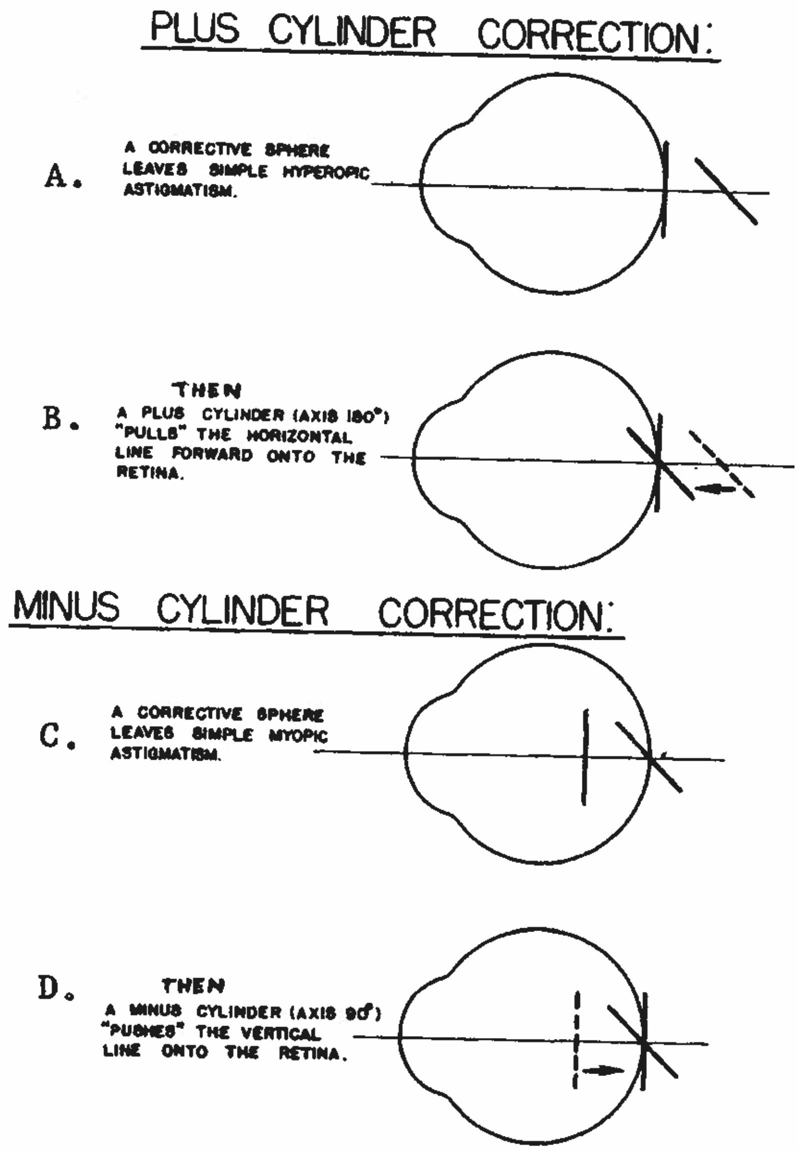
With a little practice — make up some examples yourself — it will not take you long to be able to quickly transpose corrective lenses in either plus or minus spherocylindrical form to eye *error* in the "combined cylinder form". This will allow rapid and easy access to the focal lines created by an astigmatic eye. You will soon appreciate the practical impact of your newfound skill.

We have seen once again that the spherocylindrical lens form is an equivalent, transposed expression to that given by the two separate cylinders. The latter form does allow us to picture more easily the motion of *both* focal lines as each is placed on the retina by its respective cylinder. The spherocylinder lens form can also be pictured to correct the same astigmatic error but will be shown to work somewhat differently though, of course, to the same end result.

The *spherical* part of the *correct* spherocylindrical combination will move *both* astigmatic *lines* an equal amount dioptrically — forward by plus sphere, backward by minus sphere — so that one of the two lines lands on the retina. The *cylindrical* portion moves the remaining focal line (the one parallel to its own axis) back onto the retina and so, "collapses the conoid of Sturm" and eliminates the astigmatism.

Since a plus cylinder can only pull a focal line toward itself, it should be clear that when the spherocylindrical lens is expressed in the *plus* cylinder form, the correct *sphere* moves the entire astigmatic conoid behind the retina and leaves the *anterior* line in contact with the retina; the plus cylinder then pulls the posterior line forward. (See diagram A and B below.)

A minus cylinder can only *push* a focal line back, away from itself; so, in the minus spherocylindrical lens form, the corrective sphere places the conoid entirely within the vitreous with only the *posterior* line in contact with the retina. The minus cylinder then pushes the anterior line backwards to remove the astigmatism (diagrams C and D below).



All three corrective lens forms produce the same result on an astigmatic eye; they eliminate the astigmatic error by moving the focal lines into superposition on the retina. Though each of the methods they utilize is different (as you have now seen), they all accomplish the same goal. This is what makes them equivalent.

So far, we have kept our astigmatic correction placed at the cornea. To tidy up a loose end on this subject, let's consider that the astigmatic corrective lens will be worn in the "spectacle lens plane" — a likely assumption. Its power would have to be modified to take the vertex distance into account, just as was necessary for myopia and hyperopia correction, but now separately for each major meridian. I will give one exaggerated example:

PROBLEM:

A corrective lens at the cornea is $\begin{bmatrix} +6 \times 75 \\ -9 \times 165 \end{bmatrix}$

The vertex distance is to be 15 mm.

What lens should be placed in the spectacle lens frame to have the equivalent corrective power?

ANSWER:

For the $+6 \times 75$, the far point is $\frac{1}{+6\text{ D}}$ or 16.7 cm *behind* the cornea. A corrective lens at the spectacle plane must have a secondary focal length of (16.7 + 1.5) cm or 18.2 cm. Its power must be $\frac{1}{.182} = +5.50$ D, so, $+5.50 \times 75°$ corrects one meridian.

For the -9×165, the far point is $\frac{1}{9\text{ D}}$ or 11.1 cm *in front of* the cornea. The secondary focal length of this corrective lens must be (11.1 − 1.5 cm) or 9.6 cm, and its power is $\frac{1}{.096\text{ m}}$ or − 10.4 D. A $-10.4 \times 165°$ corrects this second meridian.

The complete corrective lens placed at 15 mm from the cornea is

$$\begin{bmatrix} +5.50 \times 75 \\ -10.40 \times 165 \end{bmatrix}.$$

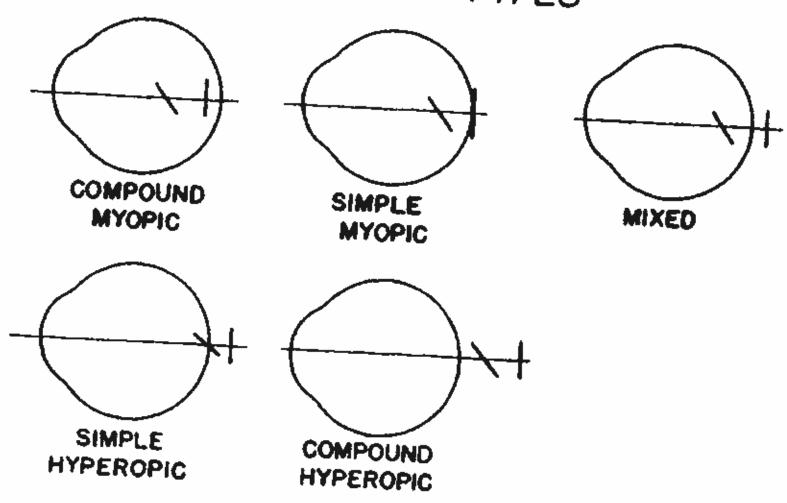
Notice that the "quantity" of astigmatism present (the difference between the meridional powers) will *seem* to vary with the position of the reference plane; measured at the cornea, it is $[+6 - (-9)] =$

15 D astigmatism, but measured at the spectacle lens plane it is [+ 5.50 − (− 10.40)] = 15.90 D of astigmatism. This astigmatism variation is only *apparent,* not real; what varies is *not* the actual astigmatic error of the eye but the amount of cylindrical *correction* required to eliminate the astigmatic error. We have already shown in our example above that this depends on the distance between the cylindrical corrective lenses and the eye's far points. This is exactly comparable to the spherical situation — the required corrective lens power varies, but this reflects its position in front of the eye and does *not* signify a change in the actual refractive error present in that eye.

Astigmatic Terms

For completeness, I simply want to define a few astigmatic terms with diagrams; they should be intuitively obvious by now anyway, but these are used frequently in conjunction with uncorrected errors or partially corrected errors. (I used them myself in the last diagram.) Actually, the only basic difference between all of them is the superimposed amount of spherical error.

ASTIGMATISM TYPES



It is clear that a *corrective* lens of $\left[\begin{matrix}+4 \times 90 \\ -3 \times 180\end{matrix}\right]$ corrects a *mixed* astigmatism since the vertical line is behind the retina while the horizontal is in front of it. But, a + 3 − 2 × 175 corrects a compound hyperopic astigmatism (both lines are behind) and a $-4 + 4 \times 33$ corrects a simple myopic astigmatism (since one line is on the retina, the other, in front). If you are unable to tell quickly just by looking at the prescription, the simplest way to discover which type you are dealing with is to express the corrective lens (or its converse — the eye error) as a "combination of cylinders" rather than spherocylindrically. In either case, if you can do this easily, it is only a manifestation of the skill you have now gained in dealing with cylinders.

"With" or "Against-the-Rule" Astigmatism

Another set of terms which are found riddled through the ophthalmic literature are "with-the-rule" and "against-the-rule" types of astigmatism. These are not very important terms *per se,* but are used so often, you should be familiar with them.

The only reason astigmatism is "*with*-the-rule" is that it is commoner than "against-the-rule" (at least throughout most of one's life; there is some tendency for advancing age to cause patients to slip over into the "against-the-rule" moiety).

The only way I can keep the terms straight is to *remember* one fact: *"with-the-rule" astigmatism is corrected with a PLUS cylinder whose axis is vertical.* (The axis doesn't have to be exactly at 90°; anywhere from 65 to 115° or so will qualify.) A $+ 4.50 + 1.25 \times 80$ corrects "with-the-rule" astigmatism.

If someone ever asks you about any relationship requiring information about "with" or "against-the-rule", you should be able to figure the problem out from the one fact I have given above.

Now, test your reasoning and your ability to utilize this one fact in answering the following questions:

Which of the following statements are *true?*

STATEMENT 1)

A $+3-4 \times 180$ corrects "with the rule" astigmatism.

ANSWER:

Expressed in the plus cylinder form, the above lens is transposed to $-1 + 4 \times 90$. Since this corrective lens has the plus cylinder correction with the axis at 90°, the statement must be CORRECT.

STATEMENT 2)

An *eye error* of $\begin{bmatrix} +3 \times 90 \\ +5 \times 180 \end{bmatrix}$ is "with the rule".

ANSWER:

This *eye error* would be *corrected* by the following lens:

$$\begin{bmatrix} -3 \times 90 \\ -5 \times 180 \end{bmatrix}.$$

This lens transposed to the plus spherocylindrical form is $-5 + 2 \times 90$. Since the plus cylinder axis is at 90°, it corrects "with-the-rule" astigmatism, so, this statement is also CORRECT.

STATEMENT 3)

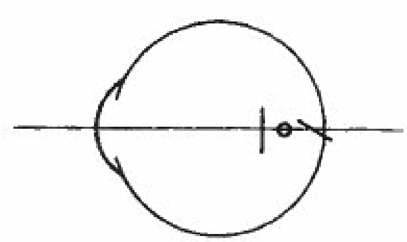
"With-the-rule" astigmatism is present in an aphake whose corneal curvature measurements indicate 43 D of power in the vertical meridian and 47 D in the horizontal meridian.

ANSWER:

These measurements show the power of the vertical meridian is too *weak* compared to that in the horizontal. This would be corrected by a lens which adds 4 D of power to the vertical meridian. A lens which does so while leaving the horizontal meridian alone is a $+4 \times$ *180* cylinder. (Remember, the power is *not* added in the meridian of the axis but 90° to it.) However, a $+4 \times 180$ corrective lens corrects *"against*-the-rule" astigmatism, so Statement 3 is INCORRECT.

STATEMENT 4)

The astigmatic line images in uncorrected "with-the-rule" astigmatism would look somewhat as follows:



ANSWER:

Here, the vertical focal line is anterior to the horizontal, and so, the plus power associated with the axis at 90° (× 90) is greater; the corrective lens for this eye must be $\begin{bmatrix} -P_1 \times 90 \\ -P_2 \times 180 \end{bmatrix}$ with $-P_1$ in diopters greater than $-P_2$; say, $\begin{bmatrix} -5 \times 90 \\ -1 \times 180 \end{bmatrix}$.

This is "against-the-rule" astigmatism (see reasoning in STATEMENT 2 above) therefore, STATEMENT 4 is INCORRECT. In "with-the-rule" astigmatism, the horizontal focal line is *always* anterior to the vertical.

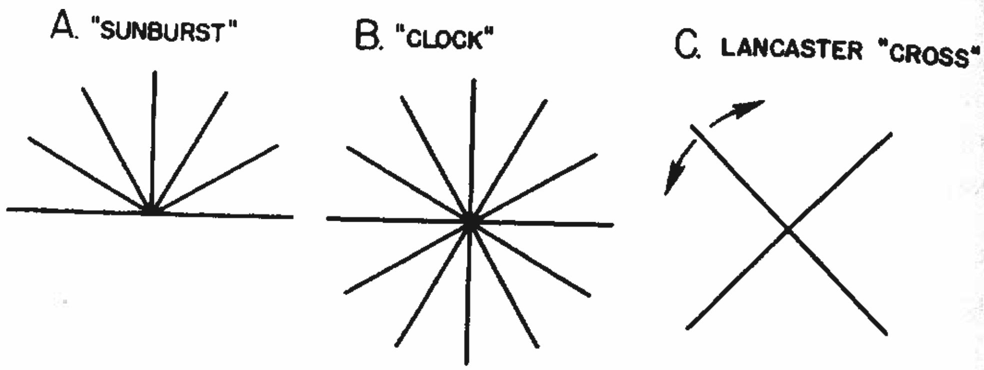
See how remembering just *one* fact about "with-the-rule" provides you with the capability of working out others?

Clinical Tests For Astigmatism

Following this exploration into the correction of astigmatic ametropia, I feel it is appropriate to slide over into a discussion of a couple of clinical tests for astigmatism. Both of these can now be completely understood. Whenever you use either or both, you should be able to picture for yourself how the astigmatic lines are positioned within the eye and how you are moving those lines with the lenses you are adding during your clinical refraction of that eye. These two tests are as follows: 1) the radial astigmatic charts and 2) the Jackson "cross-cylinder".

Radial "Sunburst" Dial and/or "Clock" Dial

These targets present a series of heavy lines arranged radially every 30°, either as a "Sunburst" or as a full "clock". (See figures A and B below.) The "Lancaster" dial is a 90° cross (figure C). This "cross" is rotatable to any angular position and so can be used in conjunction with either of the others.



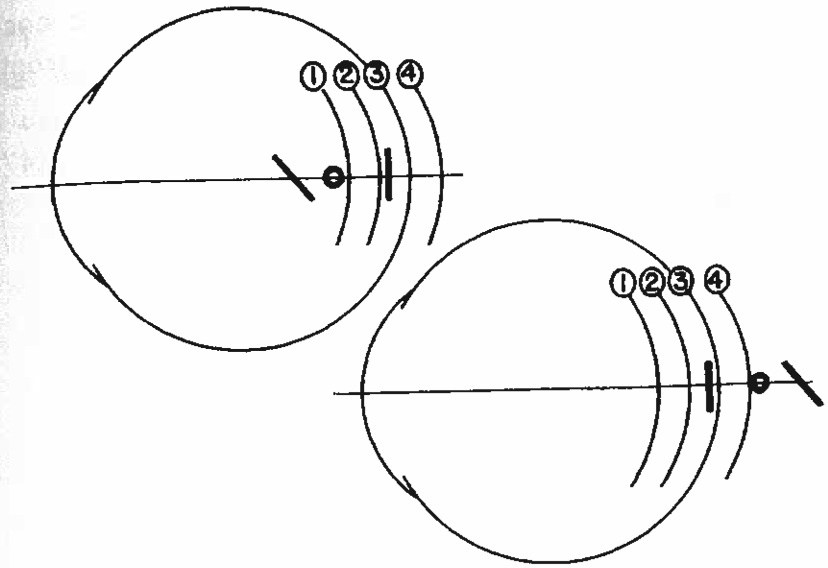
All the lines of the target are printed in equal widths with the same ink density. (The target itself is usually placed at optical infinity.*)

An astigmatic eye looking at this dial will see one of these lines as "blackest", sharpest, or clearest. That line must represent the direction of one astigmatic focal line in the eye — that particular line which is *closest* to the retina. The other line will, of course, be perpendicular to this one; but it can be either in front of or behind the retina — you cannot know which.

Let us assume it is the *vertical* line of the "Sunburst" dial which looks "darkest" to a patient. The two diagrams below show possible positions for the astigmatic line images relative to a number of different retinal positions. (Remember, each pair of astigmatic line images is created by only one of the object points making up this "dial" target.)

---

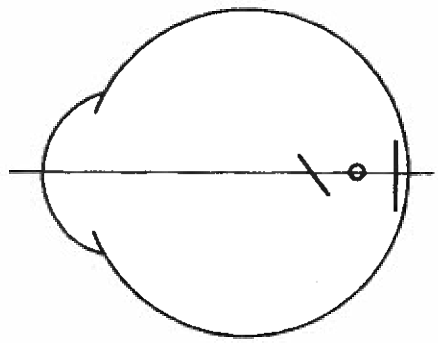
* For the human eye, a distance of 6 meters or 20 feet is considered "optical infinity", and for practical purposes does not stimulate any appreciable accommodative response. We should know that light from this distance *does* have a divergence of $-\frac{1}{6}$ Diopter at the eye; this amount, however, is considered negligible and unimportant clinically.



The vertical line will only appear *perfectly* sharp when it is *on* the retina. If it is not *on* but only clos*er* to the retina than the horizontal line, it will appear sharp*er* than the horizontal. With the astigmatic lines as shown above and the retina in *any* of the positions (1 - 4) shown in either of the two diagrams, it is the vertical line that will appear the sharper. Of course, if the circle of least confusion is on the retina, all the lines of the "Sunburst" dial will appear equally blurred.

If one or both of the focal lines falls "behind" the retina, the eye may tend to accommodate, and if it does, you may have trouble determining the true amount of astigmatic error present clinically. This difficulty — the possibility of stimulating accommodation — is gotten around very neatly in the refraction room by a technique known as "fogging".

First you will need to have some idea of the approximate correction. This you can determine as part of your routine refraction by using retinoscopy, or just by looking at the patient's old glasses, etc. (Do keep in mind that this is not a book on refraction technique, but one to allow you to understand principles.) To this approximate correction, add sufficient plus lens power in front of the cornea to make the astigmatism "compound *myopic*", that is, enough plus to bring *both* focal lines into the vitreous. Then, you know it is the posterior-most line that will be closer to the retina, and it will be in front of it. *This* is "fogging" the patient.



Obviously, the addition of plus sphere to make the eye artificially myopic can be overdone. Typically you should add only enough plus to blur the visual acuity down to 20/40 or 20/50.

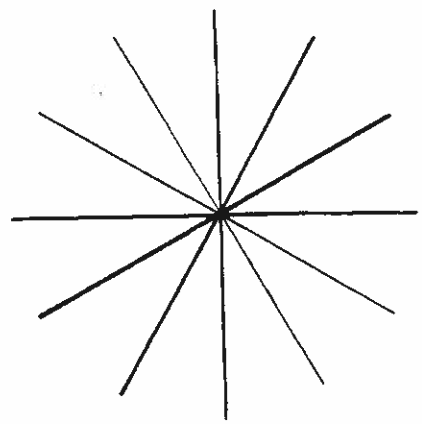
Accommodation will be suppressed by the "fog". Now when the patient says that the vertical line on the "Sunburst" target seems to be the "blackest", *you* know that it is the vertical focal line that must be the one closer to the retinal surface but, because of the "fog", you also know it is in *front* of it.

To correct the astigmatism, you must move the horizontal focal line posteriorly. What lens will do this? A *minus cylinder* × *180* (as you should be shouting loud and clear). Enough minus cylinder is added so that the patient claims all the lines look equally dark; then you know that the astigmatic interval has been collapsed, and that any blur remaining is due only to uncorrected myopia. So, at this point you put away the astigmatic dial for this eye and bring out your Snellen acuity chart. Now you can "unfog" by adding minus sphere until you obtain the best subjective visual acuity. Neat, eh wot?

Two clinical points must be made about the "dial" tests:

POINT 1)

What is the relation between the *axis* of the corrective lens you should use and the direction of "sharp lines" seen by the patient under "fog"?



Say, a patient "sees" the dial lines above where those running from 2 to 8 o'clock seem blacker than the others. Thus, *you* know the focal image line in *that* direction is nearer the retina. But, the minus cylinder you plan to use will correct the astigmatism by moving the *more blurred* focal line backwards, towards the clearer one; so, the corrective axis must be positioned parallel to the 5-11 o'clock direction since that's the direction of the line you want to move. The question is, "what *axis* is that?" To help confuse your reasoning, the patient is reading the dial like a clock from *his* position, and you, as a refractionist, are *facing* him! You can, in your mind, visualize the direction of the 5-11 o'clock line as superimposed onto the patient's eye. This direction will correspond to a meridian of 60°. (Remember the *left* ear is the 0 — 180° meridian for both eyes.) So, you should have arrived at the conclusion that the corrective minus cylinder for this patient must be placed with its axis at 60°.

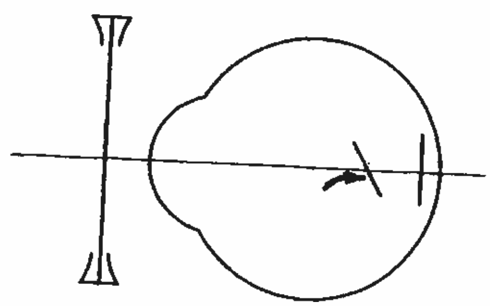
Did you get that answer? Great! Although you may have fumbled trying to convert the patient's "sharp line" response into a proper position for the corrective minus cylinder axis, you managed to come up with the correct answer by reasoning through the required steps. Hooray for you!

Though you should completely understand the above reasoning, you may find it simpler to use a little mnemonic: take the clock hour of the sharpest line (the patient's *subjective* response) and multiply it by *30°* (always 30°); *that* is the axis for the corrective minus cylinder. If the patient says the line from 4 - 10 o'clock is clearest, multiply the 4 (always use the *smaller* hour) by 30°; the corrective *minus* cylinder will be at × 120°, which you *quickly* and knowingly slip into your trial frame or phoropter.

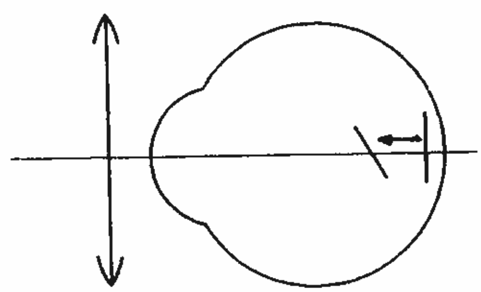
POINT 2)

The astigmatic dial charts are much easier to use if your refracting trial lens set (or phoropter) contain *minus* cylinders. In this way you are always moving the anterior (most blurry) line backward towards the retina which provides an increasing subjective clarity.

MINUS CYLINDER ×180



If instead you insist on using *plus* cylinders for this test, you are not complying fully with its intended design. A plus cylinder, which requires that its axis must be placed parallel to the line you wish to move, will "pull" its focal line toward itself, and therefore, more into the vitreous. (The plus cylinder axis will have to be held parallel to the more *posterior* line since "pulling" the anterior line forward would only increase the astigmatism.) By "pulling" the posterior focal line forward towards the anterior one, you do make the chart lines appear of equal intensity (and the end point *is* equality of *all* the lines on the astigmatic dial chart); however, here, they will all simultaneously appear to become *more* blurred. With full plus cylindrical correction, the collapse of the conoid must take place further into the vitreous than with minus cylinder correction.



To make the chart lines appear sharper (actually, to make the test work properly), whenever you add plus cylinder, you must compensate by adding *minus sphere* to shove *both* lines back toward the retina. For each 1 D of plus cylinder, you should add about 0.50 D of minus sphere. (This is called "double-clicking".)

In any case, you *can* use the Lancaster dial with plus cylinders, but it is more cumbersome than with minus cylinders. To make things even rougher for the plus cylinder user, he has no mnemonic rule for positioning the corrective plus cylinder axis, and he must either remember which subjective "clock hour" goes with which axis or he must stop to figure it out every time.

* * *

This completes the "dial" method of correcting astigmatism. Before moving to the "cross cylinder", we should expand on a subject already introduced — the *spherical equivalent.*

If you will recall, the spherical equivalent power (that focusing the "circle of least confusion" in the astigmatic image) is always halfway dioptrically between the powers of the two major lens meridians. If a lens is expressed in the "combined cylinder" form, we can find the spherical equivalent — that is, a *spherical* lens that could substitute for a toric one and maintain an image in the plane of the latter's circle of least confusion.

PROBLEM:

What is the spherical equivalent of the following lens?

$$\begin{bmatrix} +0.75 \times 90 \\ -3.25 \times 180 \end{bmatrix}$$

ANSWER:

Find the position which is dioptrically halfway between the two focal lines. This would be $\frac{+0.75-(-3.25)}{2}$ or $\frac{+4.00}{2} = 2.0$ diopters away from each line. So, the *axial* position which is 2 D away from each focal line is the spherical equivalent. In this case, 2 D posterior to the vertical focal line (positioned by $+0.75 \times 90$) is $(+0.75 - 2.00)$ or -1.25 D. *This* is the spherical equivalent for the above lens.

If a lens prescription is given in the spherocylindrical form, the spherical equivalent can be found by taking half of the (plus or minus) cylinder power and adding it algebraically to the spherical power.

EXAMPLE:

The spherical equivalent of $+1.00 - 2.50 \times 50$ is $\frac{1}{2}(-2.50) + 1 = -0.25$.

Try the following to test your own skill.

PROBLEM:

Find the spherical equivalents of each of the following lenses:

A. $+4.00 + 2.00 \times 90$
B. $-1.00 + 3.00 \times 180$
C. $-6.00 + 1.00 \times 75$
D. $-2.00 - 5.00 \times 29$
E. plano $+ 3.00 \times 50$

ANSWERS:

Spherical equivalent

A. + 5.0 D
B. + 0.50 D
C. − 5.50 D
D. − 4.50 D
E. + 1.50 D

You must be able to determine quickly the spherical equivalent of any spherocylindrical lens.

At last, we come to the second of our clinical tools to deal with astigmatism. By now, you should have acquired sufficient ammunition to use this really big gun intelligently to refine the cylindrical *power* and *axis* during clinical refraction. This weapon is the Jackson "cross-cylinder" (after Edward Jackson, the brilliant and innovative, late Denver ophthalmologist).

The "Cross-Cylinder"

What *is* a "cross cylinder?" It is a specific type of cylindrical (toric) lens that is composed of a plus cylinder and a minus cylinder of equal powers ground onto one lens, with their axes at right angles to each other. Each one of the following three *different* lenses is a "cross cylinder" — each can also be expressed in all three types of cylinder form transpositions we have studied.

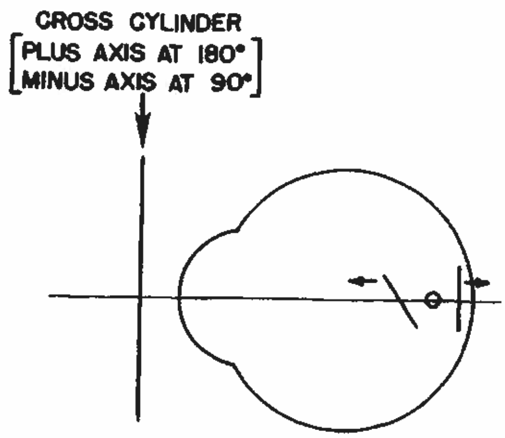
$$\begin{bmatrix} +0.50 \times 90 \\ -0.50 \times 180 \end{bmatrix}; \quad +1.00 - 2.00 \times 73; \quad -0.75 + 1.50 \times 127$$

It is easy to recognize a "cross cylinder" lens when that lens is written in the "combined cylinder" form. (See above.) However, you should not be fooled when the cross cylinder is written in the plus or minus spherocylindrical form either; in both, the strength of the cylinder is always two times and of *opposite* sign to the power of the sphere.

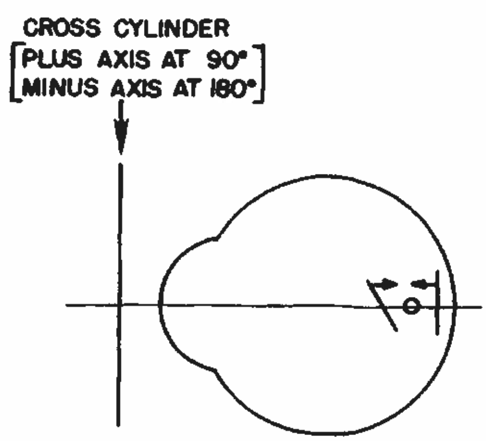
All "cross cylinder" lenses have a spherical equivalent power of zero ("plano"). If any one of these lenses is placed in front of an eye with any type of refractive error and with any amount of cylindrical error, the position of the circle of least confusion will *not* be changed. That is one of the key points in the operation of this lens as a clinical test. Now let's see how such a lens is used.

Use of the Cross Cylinder for POWER Refinement

If an eye has the astigmatic error shown below and we place a *plus cylinder* × *180* before it, only the horizontal line will be moved forward — more into the vitreous. A *minus cylinder* × *90°* in that same position would move only the vertical line posteriorly. A *"cross cylinder"* contains *both* a plus and a minus cylinder at right angles to one another, so if it is held in front of this eye with its plus cylinder axis at 180°, the minus axis will automatically be at 90° and both focal lines will be moved simultaneously away from each other, as shown below. The astigmatism will be *increased.*



Let's turn the cross cylinder so that the plus cylinder axis is vertical instead of horizontal. With this axis now at 90°, the focal line which will be moved by the plus cylinder is the vertical one, and it will be pulled forward, while the horizontal one (affected simultaneously by the minus cylinder axis now at 180°) will be pushed back. This will *decrease* the amount of the astigmatism.



NOTE: The last two figures both show the presence of "compound myopic astigmatism". This is *NOT* where the focal lines *should* be for the proper performance of this clinical test, as will soon be explained. The lines and circle of confusion are shown this way for diagrammatic simplicity only. The circle *should* be on the retina, not in the vitreous.

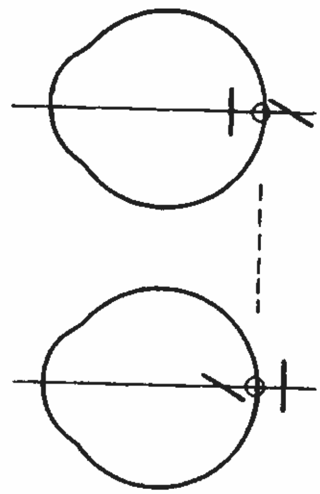
In any case, the *size* of the circle, which represents a single object point, is dependent (among other factors) on the amount of astigmatism present. When the circle is on the retina where it belongs and the eye is studying a line of Snellen letters on a distant chart, the addition of a cross cylinder with its plus axis at 180° would cause that line to blur subjectively since the *total* astigmatic error is *increased* by it. (Even though the plane of the circle of least confusion stays put, each circle obligatorily increases in size as the astigmatism increases.) But with the plus cylinder axis turned to 90°, the size of the blur circle is decreased; this increases the subjective clarity of a line of Snellen letters located in the distance.

Clinically, you have to ask the patient which of the two cross cylinder positions provides the better, clearer image. When he tells you, follow the directions provided by the cross cylinder itself. Slip the proper *cylindrical* lens into your trial frame or phoropter. An example follows:

The patient tells you that the Snellen letters seem to be clearer when the plus cylinder axis of your cross cylinder is at 90°. Thus you know that the cross cylinder in that position must have reduced the amount of original astigmatism. This directs you to add more plus cylinder with its axis at 90° to your trial lens. (If you happen to be using *minus* cylinder trial lenses, follow the direction given by the *minus* cylinder axis of your cross cylinder, and add or subtract *minus* cylinder from your trial lens appropriately.)

You continue to follow the "instructions" given you by your cross cylinder, adding or subtracting cylindrical lenses as directed, until the patient notices *no difference* between flips of the cross cylinder. As soon as the patient is unable to detect a difference between the two positions of the cross cylinder, you have totally "collapsed" the previously uncorrected astigmatism of the eye. The only astigmatic error now present in the system is that induced by the cross-cylinder lens itself.

The combined figure below shows the optical condition of "equality" between "flips". The two focal lines created by the cross cylinder itself are shown; the circle is *on* the retina. When the patient proclaims that subjective "equality" is present, the focal line positions have simply been interchanged.



CLINICAL POINTS:

If you continue to add only plus or minus *cylinder* as directed by the cross cylinder, you *will* be collapsing the conoid, *but* you will also be moving the circle of confusion by moving only *one* of the two focal lines. For every 0.50 D of cylinder you do add, you will simultaneously move the circle of least confusion 0.25 forward or back, depending on whether you are adding a plus or minus cylinder. So for practical purposes, you must compensate for the shift in the circle's position by adding a 0.25 Diopter of *sphere* of *opposite* power every time you introduce 0.50 D of new cylinder. Thus, if you add + 0.50 cylinder × 90, you should also add − 0.25 D sphere to maintain the position of the circle of least confusion in its original position — hopefully on the retina.

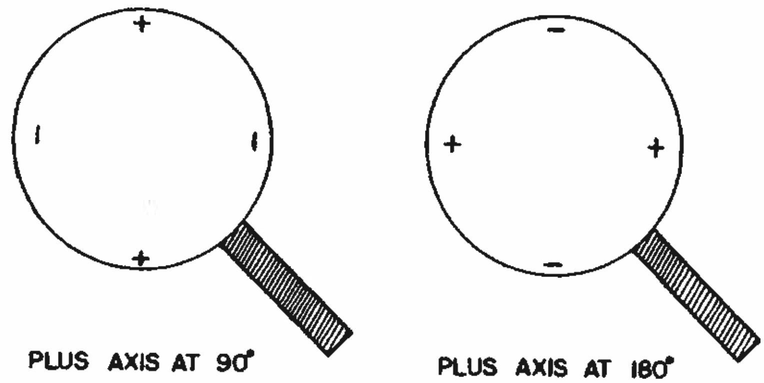
At the "equality" endpoint, we have shown that there *is* astigmatic error present with the cross-cylindrical lens in place; an alert patient will frequently remind you that he sees better *without* that lens. Because you always would like patients to believe that *you* have the upper hand and know what is going on, it is better to warn them ahead of time; "I realize that the lens I'm about to introduce may blur the target letters somewhat, but all you have to tell me is *which* lens position is the *better* — one or two!" (meaning one of the two flipped positions).

Another refraction hint: Whenever giving patients a choice between any two different lenses but especially when using the cross cylinder, always make the lens switch abruptly and cleanly. This will make the comparative evaluation much easier for the patient. You should shift between the two positions quite rapidly and allow the patient no longer than *one second* at each position to decide.

Typically, clinicians become squirmy and impatient while the examinee sits trying to make up his mind (the patient isn't very comfortable either)! You really simplify his task by forcing him to choose rapidly. Tell him you know you're changing the lenses quickly, but what you want from him is a quick, "first impression" and *not* a carefully thought out, deliberate, judgment decision. So, "push" him and "lean on" your patient for a quick response while reminding him that there is no "wrong" answer.

You may wonder why I dwell so much on this simple point. The reason is that only rarely have I found it performed correctly, and any incurred inaccuracy is usually blamed on the *test!* Performed properly, it will not only speed up your refraction, it will also make it more accurate. But please, temper this instruction with common sense. Older patients tend to be "slower" in general so you will have to adjust your pace somewhat, but even then, keep "pushing". By giving them time "to think", you are only allowing them the opportunity to adapt to the stimulus; this will only make their selection *more* difficult.

The rapid transfer in position of the plus axis and the minus axis is facilitated by the construction of the cross cylindrical test lens. An arm is mounted at 45° to the two principal axes. On the standard cross cylinder, the meridians marked are the *axes;* the plus is indicated with a small, white plus or a white dot; the minus is marked in red.

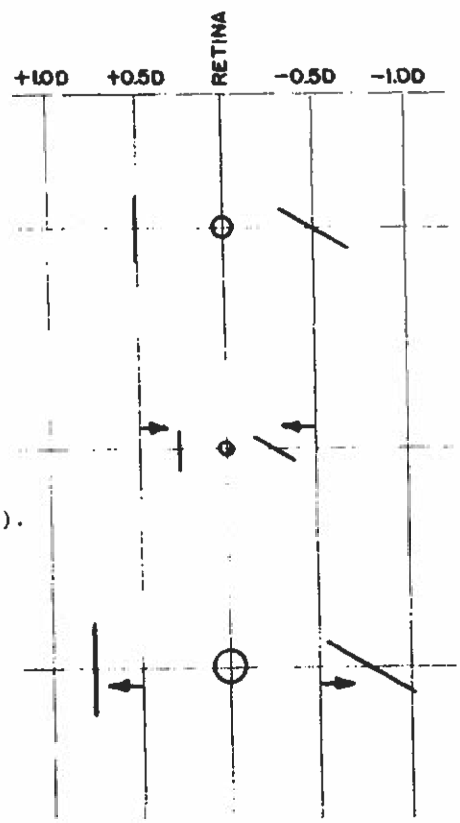


A rapid rotation of the knurled arm between the examiner's fingers quickly interchanges the plus cylinder axis and the minus cylinder axis.

Another tip which you may also consider superfluous but, performed well, will speed up your refraction and reduce frustration all around. You will greatly facilitate your patient's choice of one lens position over another by your giving a "name" to each one — calling it "one or two", "A or B", etc. This immediately identifies for the patient which lens position you are asking him about. I've heard even experienced refractionists say "which is better — *this* lens or *that* one", leaving about 3-4 seconds for a choice. During this prolonged period of time, the patient and the doctor lose communication and neither knows which position "*that* one" is! It will make no difference whatsoever if you rechristen the same lens with a new name during the next sequential presentation. Lens 2 can become lens 3 in the next paired choice and thus will not be confusing for the patient. So, make it easy for both of you; *name* each lens position and make sure you do so *simultaneously* with each lens presentation, *every* time you flip it back and forth.

I would like to demonstrate that when you flip a cross cylinder between its two positions you create an astigmatic *change* of *four* times the power of the cross cylinder; that is, a .25 cross cylinder (− 0.25 + 0.50 × 90) will cause *1.00 D* of astigmatic *difference* between positions "one" and "two" of the cross cylinder, no matter what the eye's refractive error.

First, look at an eye with an existing error of 1.00 D astigmatism:



With an eye that has an uncorrected cylindrical refractive error of + 1.00 D (the separation between focal lines is 1.0 D), use appropriate sphere to shift Sturm's interval so as to position the circle of least confusion on the retina.

POSITION 1
Place a 0.25 D cross cylinder (+ 0.25 - 0.50 x 90) in front of above eye: each line will be moved inward by 0.25 D; the total astig. error remaining is now only 0.5 D and the circle of least confusion shrinks (as does each line).

POSITION 2
Flip the above cross cylinder to (-0.25 + 0.50 x 90): each line moves outward. (Compared to the lines in Position 1, each line has been moved out by 0.50 D). The total astigmatism here is 1.5 D and the circle of least confusion enlarges (and the lines elongate).

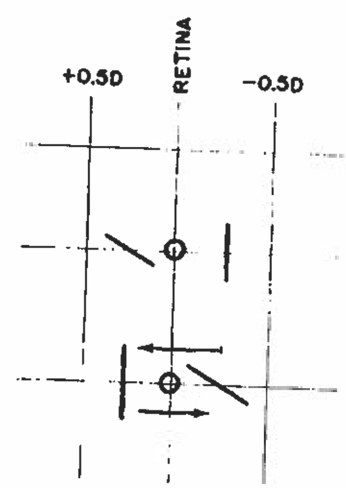
Thus, the *difference* in astigmatic error between POSITION 1 and POSITION 2 is (1.5 D − 0.5 D) = 1.00 D. Q.E.D.

If instead the eye is *emmetropic*, it is a bit harder to "see" that this much shift still occurs, though it really does. The 0.25 cross cylinder introduces only + 0.50 diopter of cylinder in each position. Flipping the cylinder simply switches the positions of the focal lines and so does not "seem to change the *quantity* of astigmatism; there *is* only .50 D present. Realize, however, that since the lines are interchanged, each must have moved by 0.50 diopter, so, there must have been a "change" of *one* full diopter (adding together the two line movements). Look at the diagram sketch below:

Place the 0.25 cross cylinder before an eye which has *no* cylindric eye error:

POSITION 1
(+0.25 - 0.50 x 90)
places the focal lines as shown:

POSITION 2
(-0.25 + 0.50 x 90)
switches the focal line positions as indicated.
The circle of least confusion remains the same size.



Although the astigmatism present in each of these two positions is only 0.50 D, the algebraic *difference* between them is + 0.50 − (− 0.50) = + 1.00 D. This difference is not as "visible" as in the previous example with a pre-existing astigmatic error; yet, it is there nonetheless. So, *any* cross cylinder will induce a shift of four times "its name" between its two flipped positions.

CLINICAL POINTS — Practical hints for the use of the cross cylinder:

1) The cross cylinder is not used to make a rough, initial approximation of the amount of astigmatic refractive error present; it *is* used to *refine* a close approximation to the existing error.

2) Before starting to use the cross cylinder to refine the cylindric corrective lens *power*, have the patient's *best* visual correction (sphere *and* cylinder) in place. Have him able to read the *smallest* line of the acuity chart that he can — 20/20 if possible, while using the closest you can get to his "full" correction. If you are in doubt as to whether or not you should add another −0.25 D of sphere to sharpen the patient's vision before using the cross cylinder, *do so*. It is better to have him slightly overcorrected in minus than to have him myopic. It is quite *incorrect* to "fog" or blur the vision before proceeding with the cross cylinder refinement. I say this in spite of the fact that "fogging" was proposed for *this* test by none other than Jackson, the inventor, himself. (Even great men do make errors — he *was* wrong here). If you *do* have the patient "fogged", he will have more tendency to vacillate in his responses and switch "back and forth".

3) After making sure the patient's *best* refraction is in place, use the cross-cylinder with the patient looking at a line of Snellen target letters about 1-2 lines *larger* than his threshold acuity line. If the maximum acuity is 20/25, use a 20/30 or 20/40 line for the cross cylinder test. You want to present *almost* threshold letter size, since with this type of challenge the patient's judgment is more critical. But you can't use the *smallest* letter size since the introduction of the cross cylinder itself, even in an emmetropic eye, introduces its own cylinder error (a + .50 − 1.00 x 180 cross cylinder introduces 1.00 D of cylinder) and this astigmatism will blur his retinal image too much. I would suggest that the refractionist use a cross cylinder of the *least* power possible, gauged for the maximal acuity level. (I have already pointed out that the cross cylinder is *named* for its spherical component. The above lens is called a *.50 D* cross cylinder; a −0.12 + 0.25 × 72 is a *.12 D* cross cylinder, etc.) See the following chart for the appropriate cross cylinder lens to use:

Maximal corrected visual acuity	*Size of cross cylinder to use*
20/15 — 20/20	.12 D
20/25 — 20/30	.25 D
20/40 — 20/50	.50 D
20/60 — 20/100	1.00 D

For 20/200 and poorer acuities, cross cylinder refinement is not important and probably a waste of effort.

4) For the cross cylinder to help you refine the astigmatic *power* of a corrective lens, the cross cylinder axes (and the corrective cylinder) *must* be aligned parallel to the primary astigmatic meridians of the eye; you must be "on axis". Only then can the "cross" cylinder move the focal lines predictably as described. We will soon see how you make sure the axes *are* correct.

* * *

You now know how to refine the corrective cylinder *power* with a cross-cylinder. We still have in our bag the other beautiful application for this instrument — the refining of the corrective cylinder *axis.* Though the practical techniques for the two determinations will look different, they are in fact based on optical principles which are really very closely related. These principles pertain to how cylinders will add together when superimposed.

I explained the optics of the *power* check first because this is simpler to understand. But, clinically, as I have already cautioned, the proper use of the *power* check depends on good alignment between the cross cylinder *axes* and the major astigmatic meridians of both the corrective trial lens and the patient's eye. Thus, the check for correct *axis* using the cross cylinder must be done first, prior to checking the *power.*

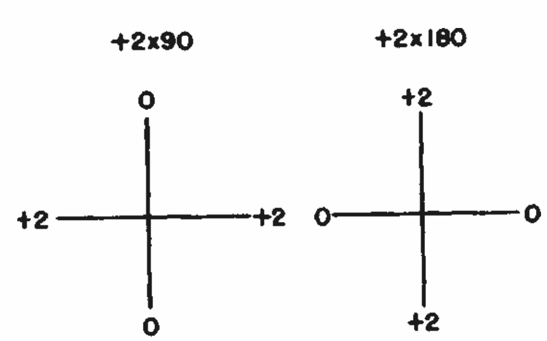
Now, to complete your understanding of how the cross cylinder can be used to refine the corrective *axis,* you will need to examine more closely what happens optically when two cylindrical lenses are added. Return with me now to more of the "basics".

Addition of Two Cylinders

When two cylinders are superimposed, the resultant optical effect is not *always* obvious. It *is* easy, however, to visualize what happens when their axes are *aligned*, parallel to each other. In this case we have already seen that the principal meridian powers can simply be added algebraically; a $+ 2 \times 90$ added to a $+ 2 \times 90$ combines to give a $+ 4 \times 90$; a $+ 2 \times 180$ adds to a $- 4 \times 180$ to yield a $- 2 \times 180$. It is quite a bit more touchy when the principal meridians are not parallel but misaligned by some angle. First, let's consider the situation when the two cylinders are of *like sign* (when both are plus, or both are minus cylinders).

At first glance it might seem that the cylinders would add "normally", like physical vector forces which act at some angle to each other — that is, a $+ 2 \times 90$ and a $+ 2 \times 80$ combine to yield a $+ 3.8 \times 85$ — the axis being midway between the two when the powers are equal. This is true, but *in addition* something unexpected happens; as if out of nowhere, some plus sphere is also produced!

The greater the angle between the two like cylinders, the greater the amount of sphere induced and the *less* the resultant cylinder (the *axis* always being half-way between when the two cylindric lenses are equal in power). This increase in sphere and decrease in cylindrical power continues as the axes are rotated until they are fully 90° apart. At that time, . . . well, look at our crosses (in the figure below) — we have $+ 2 \times 90$ added to a $+ 2 \times 180$:



As we can see, when the axes of the like and equal cylinders are crossed at 90°, the resultant is a + *2 sphere!*

So, with the above example, as the angle between the two axes varies from perfect alignment (when the resultant sum is *plano + 4 × 90*) to where they are 90° apart (yielding a resultant of + 2.0 D sphere), there will be a progressive increase in sphere and decrease in cylinder power. But please note, at "in-between" axes the exact amount of resultant sphere and cylinder is *not* easy to obtain. At 45° separation of axes for example (with one axis at 90°, the other at 45°), the resultant is 0.6 + 2.8 × 67.5°.

As you might guess, it gets more complicated rapidly with more complicated spherocylindrical additions. There *are* complex formulas written which can be used to calculate the resultant obtained when *any* two lenses are superimposed at *any* angle. (An easier, graphical method is also available). However, both the graphical and mathematical methods are so cumbersome that I gleefully state they are relatively unimportant for us — since we *do* have an "out"! Every clinic refraction room has a lovely "computer" in the *lensometer,* the everyday instrument used to determine the power and axes of spherocylindrical lenses. This gadget can instantly determine the resultant power and resultant axes of any two (or more!) lenses in combination. Just slip them together into the lensometer, clamp them in place and read the instrument as if there were only one lens in place — the proper resultant will be given to you in a flash without complicated formulas or graphs. Why learn something you'll never use when you have at hand the means to solve such problems readily? The lensometer provides the easiest, most pragmatic way to obtain a quick and accurate answer, and the simplest way has to be considered the most *scientific* way to do so.

So, we have just looked at combining cylinders of *like* sign when the axes are misaligned. It is even more surprising to find what happens when two cylinders of *unlike* sign are combined "off" axis. Take a practical example:

When you fully "correct" an eye with a built-in astigmatic error of − 2 × 80°, you should use a + 2 × 80° spectacle lens. What if you mistakenly put the corrective lens at axis 90° instead of at 80°

where it belongs? You would *not* obtain a resultant with its axis between the two; you unexpectedly induce a new spherocylindrical combination with its plus cylinder axis located 45° to the "bisector" (85°) axis! That is, the plus cylinder axis of the new resultant is at 130°, (the minus cylinder axis would be at 40°). So you see, the new resultant obtained by misjudging the eye's real axis by only 10° is $-.35 + .70 \times 130°$. (For this eye, this resultant is a new "error" which also would require correction).

What you as a clinician should remember is that the *plus* cylinder axis of the *resultant* error is 45° to the "bisector angle"; but in which direction is that? It is always found on the *same side* of the "built-in" eye cylinder axis that you have placed the axis of the plus cylinder of the "corrective" lens. "Come again?" you say.

Example: An eye's "built-in" minus cylinder axis is at 130°; a "corrective" lens *plus* cylinder axis is *mis*placed at 125°; the resultant *plus* cylinder axis will be at 82.5° (in the same direction as the erroneous placement of the plus cylinder). The minus cylinder axis, of course, will be on the other side — at 172.5°.

With this background on how cylinders act when superimposed, we can finally understand how the cross cylinder can be used to refine the *axis* position of the corrective cylindrical trial lens. (Remember, to allow you to achieve optimal results from the cross-cylinder when refining either power *or* axis, do not have any "fog" present).

Use of the Cross Cylinder for AXIS Refinement

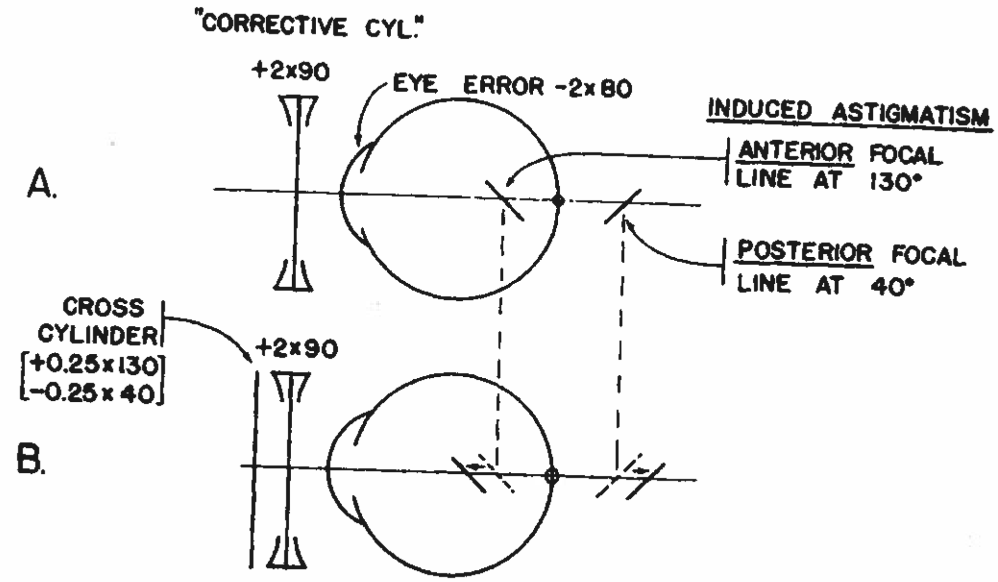
Instead of aligning your cross cylinder axes *parallel* to those of the *corrective* lens cylinder (as you did to check on the "power"), you must align the cross cylinder axes parallel to the induced *resultant* axes. As pointed out above, these resultant axes are approximately at 45° to the axis of the corrective trial lens you are using.

To reiterate: 1) there *will* be a *resultant* cylindrical error present whenever your corrective trial cylinder is *not* perfectly aligned with the eye's refractive error, and 2) the *resultant* astigmatic error will produce focal line images. To move those line images toward or away from each other (as you did in "collapsing the conoid" when you refined the *power* of the corrective cylindrical lens), you must place the cross cylinder axes *parallel* to those resultant focal lines. This will be approximately accomplished if your cross cylinder axes straddle (at 45° to) your correcting cylinder axis.*

Let's take an example: Assume that the eye shown below has an *error of* $-2 \times 80°$. The "correcting" lens is misplaced 10° "off" the proper axis, that is, the + 2 cylinder is placed with its axis at 90°. The "induced error" due to this 10° misplacement is $-0.35 + .70 \times 130$. (Notice that this error itself is a "cross-cylinder"!) This *resultant* error is equivalent to the following "combination of cylinders":

$$\begin{bmatrix} + .35 \times 130 \\ - .35 \times 40 \end{bmatrix}$$

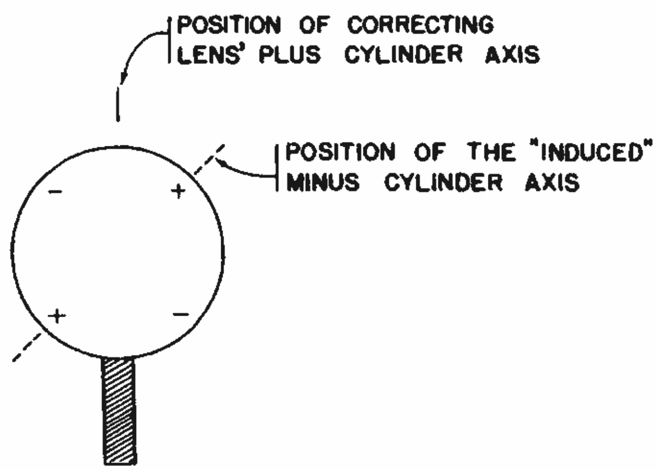
Therefore, you can easily see that the focal line which is oriented at 130° is in vitreous (since it is pulled forward by the + 0.35 D power), while the line at 40° is pushed "behind" the retina.



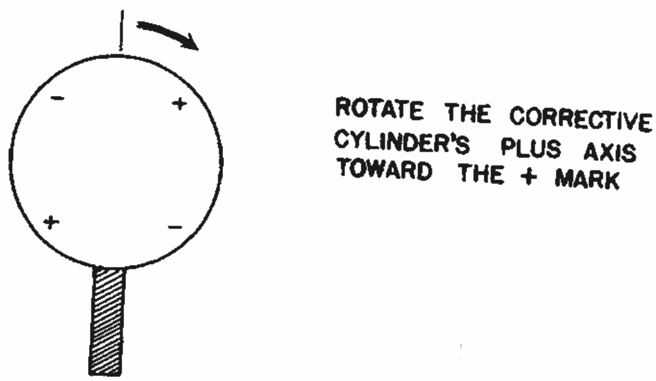
* Theoretically, you should not straddle the axis of the *correcting* cylinder but of another meridian; that latter meridian is located somewhere *between* the correcting cylinder's axis and the axis of the "eye error". However, in clinical practice, this distinction is *not* important. (Actually, it is only the position of the "correcting" cylinder axis that is firmly known anyway.) The diagram on p. 183 shows the typical *clinical* straddling position and not the theoretical one.

If you now place your cross cylinder's plus axis parallel to the *induced* plus cylinder axis, the meridional powers will, naturally, add directly. The cross cylinder $\begin{bmatrix} +0.25 \times 130 \\ -0.25 \times 40 \end{bmatrix}$ will "pull" the 130° line (already in the vitreous) further into the vitreous, and "push" the line at 40° further back; this *increases* the astigmatism present, enlarges the circle of least confusion, and further blurs any target letters the patient sees. (This situation is diagrammed in figure B above.)

When the cross cylinder is "flipped" 90° to that shown in B, the astigmatic focal lines will then approach each other, decrease the astigmatic error, and the patient will exclaim that this second position of the cross cylinder allows clearer vision. This subjective improvement occurs, then, whenever the "cross-cylinder's" *plus* axis falls on the induced *minus* cylinder axis; this tends to neutralize the induced error. Now the examiner should look at his cross cylinder and note the exact position of its *plus* axis, since the latter indicates that the induced *minus* cylinder axis is also at the same orientation (here, at 40°). What the examiner will see when *facing* his patient (who now "sees better") is the cross cylinder in the position shown below; the plus axis is superimposed on the *induced* minus cylinder axis.



Since we now know that there indeed *is* an induced minus axis we can conclude that there must have been some misalignment of the original corrective lens' *plus* axis to produce it; in addition, we also know that the corrective lens' axis must have been positioned too far on the *other* (counterclockwise) side of the axis of the "built-in" eye error to put the induced minus axis where it is. So, in order to improve matters, we must rotate the *corrective* lens' axis in a *clockwise* direction, that is, *towards* the plus cylinder mark on the cross cylinder lens.



You obviously do not have to reason this out every time you do the test; just rotate the corrective lens' *plus* axis towards the *plus* axis mark on the cross cylinder whenever the patient tells you that particular cross cylinder position is best. If you are using a *minus* cylinder corrective lens, you will have to rotate that lens towards the indicated *minus* cross-cylinder axis.

Continue to refine the position of the corrective cylinder axis by flipping your cross-cylinder back and forth, while straddling each *new* axis, as the patient tells you how the clarity between the two flipped positions compares. You should continue "flipping" until the patient notices "no difference" between the two positions of the cross cylinder. When this occurs, the correcting lens cylinder axis will be properly aligned with the built-in eye cylinder, and the only cylindrical error now present will be that created by the cross cylinder lens itself.

In using the cross-cylinder, remember to check for *axis* refinement first, then check for *power;* and then, if you really wish to be accurate, recheck the axis again.

I have devoted quite a bit of space on the cross cylinder and its use. You should be able to judge the commensurate esteem in which I hold these magnificent tests!

I would like to tie together the subjects of astigmatism, the "stenopeic slit" and the "pinhole" — a clinical screening test for refractive error. Try as I might, I am unable to explain it any more clearly than I did in a previous article*; I am therefore reprinting it here.

An Elongated Pinhole — The "Stenopeic Slit"

On beginning an optics conference last year with a prominent residency group, I mentioned casually that among the subjects to be covered during that conference would be the principles behind the use of the stenopeic slit. "The what?" was the general reaction. What really surprised me was that some of the more highly trained residents had been taught neither how to manipulate this useful tool nor even what it *looked* like, even though most of the residents had at times wondered what *was* "that peculiar-looking disç" lying around in a trial set. Because of this experience, I felt that it might be instructive to review here the use of the *pinhole* and then go on to study the *elongated pinhole* and its usefulness in arriving at an estimate of cylindrical refractive error.

Pinhole

In emmetropia, every point in an object of regard is brought to a point focus on the retina. Thus, if one neglects the effect of diffraction, there is a *point-to-point* correspondence between object and image. The *sum* of all point foci yields a uniformly sharp retinal image of the object being scrutinized. If a refractive error exists (any kind of error), a "blur circle" is formed on the retina instead of a "point." The size of that blur circle is directly proportional to the size of the subject's pupil.

---

* Rubin, M. L.: "The Elongated Pinhole" Survey of Ophth. 13:355-359.

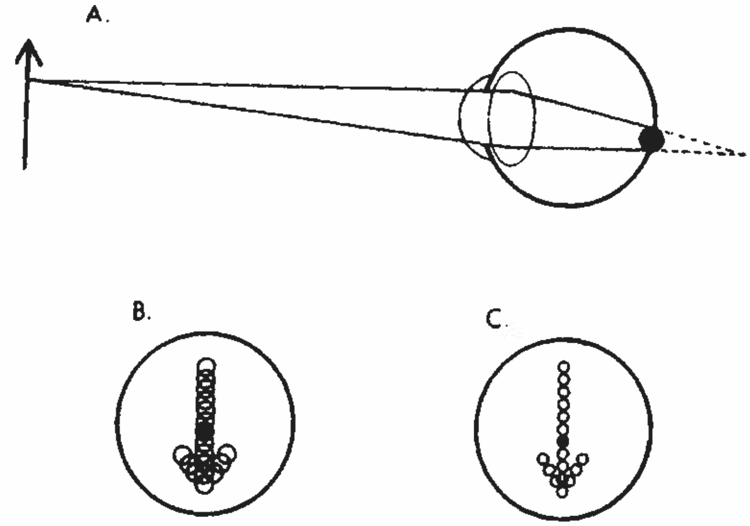


FIG. 1. *A*, light rays from each and every point on an object form a blur circle on the retina of a hypermetropic eye. *B*, the retinal image in this eye is the composite of all blur circles, the size of each being proportional to the diameter of the pupil. *C*, if a pinhole is held in front of this eye, the blur circles are decreased in size, thus sharpening the over-all retinal image.

In Figure *1A*, light from every point on the object is brought to a focus behind the hypermetropic eye, since its refractive power is too weak. What *is* on the retina is a blur circle, or a series of blur circles when the entire image is considered (Fig. 1*B*). If the pupil were made smaller, the diameter of that blur circle would decrease, and the retina would "see" an image more like the correct "point image" (Fig. 1*C*). If a pinhole aperture were used immediately in front of this eye, it would act as an artificial pupil and the size of the blur circles would be correspondingly reduced. This is the reason that many ametropic individuals (especially myopes) squeeze their eyelids together to see more sharply, since in this manner they narrow their pupillary apertures.

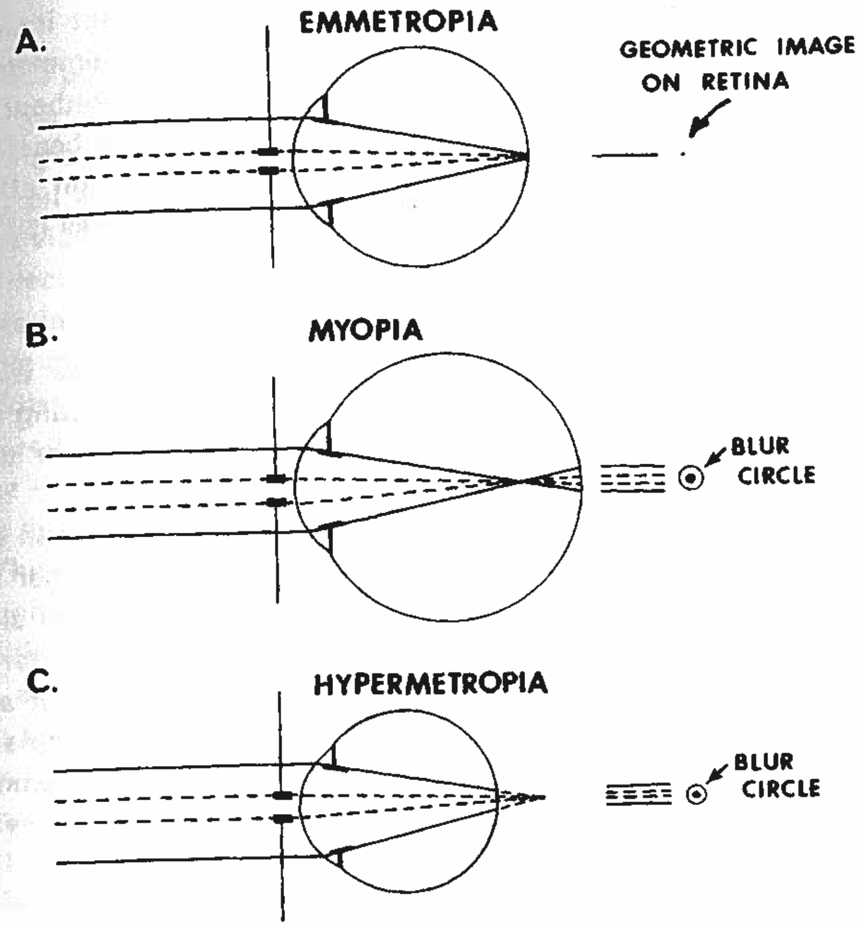


FIG. 2. The effect of a pinhole introduced in front of an emmetropic eye, a myopic eye and a hypermetropic eye, respectively (neglecting the effect of diffraction). The pinhole has no effect on the emmetropic eye *A*, but in both *B* and *C* the blur-circle size on the retina is shown to decrease with the pinhole aperture.

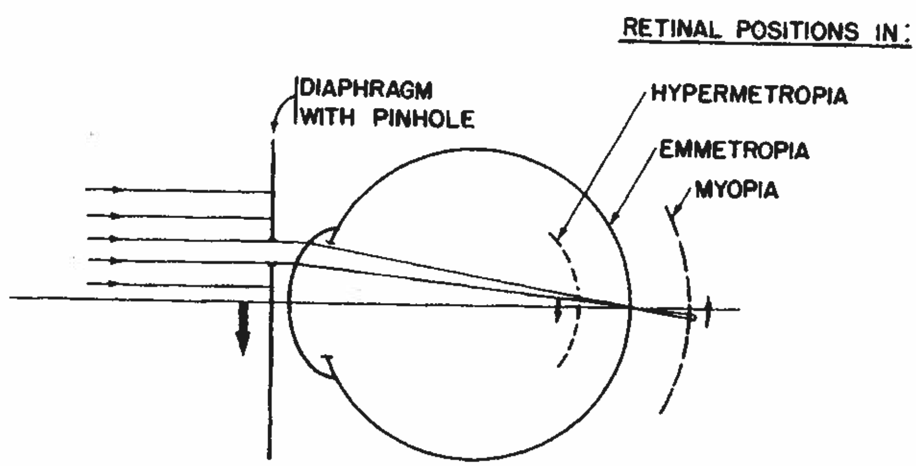
Figure 2 demonstrates the effect of the pin hole in an emmetropic eye, a myopic eye and a hypermetropic eye. This figure shows the comparable sizes of the retinal image of a point source with and without a pinhole held in front of the cornea.

Although the pinhole actually decreases the total amount of light falling on the retina, it does create a smaller blur circle when there is any ametropia existing, and thereby improves the sharpness of the retinal image. However, the pinhole itself does induce a diffraction effect also, and thus tends to blur slightly the sharp point imagery normally present in emmetropia. (The *best* acuity in an emmetrope *with* a pinhole is rarely better than 20/25 even though, without the pinhole, vision may be 20/15). In ametropia, however, the beneficial effect on blur circle size certainly outweighs the diffraction effect. In summary, the pinhole "pinholes" the blur circle and makes it more like a "point."

* * *

Here, I would like to interject with two other phenomena which were not mentioned in the original article; these are so interesting that you as clinical students of optics ought to be made aware of their existence.

The pinhole described above will not only improve acuity in any type of refractive error but can also tell the examiner whether the refractive error is *hyperopic* or *myopic!* If the pinhole is moved side-to-side in the frontal plane while the patient is observing some target through the hole, the ametropic patient will note that the *target* also appears to move — the same direction as the pinhole in myopia or the opposite way in hyperopia. *No* movement will be evident if emmetropia is present. The optical basis for this is shown in the following diagram:



The *sharpest* image point created by the pinhole will be in the position where the emmetropic retina would be located, that is, in the "emmetropic" focal plane. As the pinhole is moved *down* across the pupillary opening, the small "blur circle" which would be found on a hypermetrope's retina will move in this *same* direction; this is visualized by the patient as motion opposite to (or "against") the direction of the pinhole motion — vice versa for the myope. (The arrows in the above diagram point to the directions of motion of the pinhole and the *retinal* images.)

What I have shown here in ametropia occurs with the tiny, circular retinal image of a single object *point;* it is clear that the *entire* image — a composite of many image "circles" — will shift in the same direction as the individual "circles" do.

This subjective test can also be used to locate and measure the near point of accommodation. The patient holds a test card up close and tries to read small print while looking through the "moving" pinhole aperture. If he can indeed accommodate for a given distance, there should be no subjective appreciation of any motion of the target when the pinhole moves. If there is, the target is too close, and should be moved back until no "motion" is noted; in that position, the near point will have been located.

These are actually quite sensitive subjective tests and can detect from 0.25 D to 0.50 D of error.

Another type of motion induced by the pinhole in an ametropic patient creates one more subjective test for myopia vs. hyperopia. This one is not quite as sensitive as the former but is still quite vivid.

Here, the pinhole is moved towards-and-away from the patient while he continues to view some target. On moving it away, if the error is uncorrected myopia, the target will visibly *shrink* in size; if it is undercorrected hyperopia, it will seem to expand. There will be no size change if a full "correction" is in place.

The optical explanation of this revolves about the fact that the pinhole functions as a *universal* corrective lens, and we will soon see that corrective lenses induce magnification or minification, the respective effect being greater as the corrective lens is positioned further from the eye. (In hyperopia, retinal images are magnified as the corrective lens is positioned further from the eye; in myopia, the retinal image is minified.) The pinhole mimics this "lens effect" exactly (and for the same reason) as it moves to and from the eye. This finding as it pertains to lenses will be expanded upon later.

Another interesting phenomenon in a similar vein can be demonstrated using the pinhole. This also is based optically on the fact that the pinhole serves as a *universal* lens:

Hold a pinhole close to your eye. Also hold up any plus lens, say a + 4 D sphere at about 25 cm, and look through both. You will have created a small Galilean telescope of about 4 × magnification power with the pinhole as the "eyepiece". Now, if you watch closely while you move the + 4 D lens further away, you will observe a continuously increasing magnification, like a zoom telescope! The pinhole serves as a continuously variable-minus eyepiece lens. (The complete "lowdown" on telescopes later.)

CLINICAL POINT:

For any particular patient the pinhole makes for a good, quick check on whether or not it is refractive error *per se* that is the cause of a decreased acuity; and, as already explained, because of the blurring effect of *diffraction*, an accurate lens-correction of the refractive error will almost certainly yield a better acuity measurement than the pinhole itself provides without the corrective lens.

Although this is the general rule, the alert student should be aware of exceptions. Sometimes, vision through a pinhole is *much better* than can ever be obtained with corrective spectacles or contact lenses. This finding might even be *expected* in some patients with keratoconus, or those with discrete, small opacities of the cornea or lens, or in cases of irregular astigmatism (when the major meridians are not at right angles to one another) as in a distorted but clear cornea warped by a stromal scar. In such patients *without* the pinhole, the refraction occurring at the surfaces may be irregular and non-predictable, and if any opacities are present, they can cause scattering — all these effects destroy the possibility of sharp retinal imagery. However, with a properly positioned pinhole, a patient can select a small clear area through which light can pass, unimpeded by any ocular obstacles, and thereby create a good quality of retinal image. Thus, despite diffraction, in these patients, vision through the pinhole will be better than without it. (Do keep in mind that this appliance will *not* offer acuity improvement to a patient with a *diffuse* opacity of the transparent media, such as in corneal epithelial edema, mature cataract, vitreous haze, etc.)

Now, back to the reprinted article.

The Stenopeic Slit

To understand the basics of the stenopeic slit, the fundamentals of astigmatism must be kept clearly in mind. Study the diagram in figure 3*A* which shows a cylindrical lens grasping a bundle of incident light and bringing it to an a-stigmatic (nonpoint) focus, that is, two line foci with Sturm's interval between.

The length of the astigmatic focal line images is also directly proportional to the size of the aperture as well as to the *amount* of astigmatism, and inversely proportional to the total spherocylindrical power influencing any given line. Let us, however, restrict our attention to the influence of the aperture size. If we now cover up part of the lens so that we narrow its vertical dimension (Fig. 3*B*), the vertical line is decreased in length while the horizontal is not affected. If we further narrow the vertical to a horizontal slit-like opening (called the *stenopeic slit*), the vertical line will be "pinholed" as shown in Figure 3*C;* the length of the horizontal line still is uninfluenced.

With the slit in the horizontal position, then, there is a position of the image plane (retina) where there is practically a point-for-point representation for every point in the object, and hence a position of sharpest imagery. A patient who has his retina located at this position will have a "sharper" image than one who has his retina located anywhere else behind this lens. If the slit is turned vertically (Fig. 3*D*), it is the horizontal line which is "pinholed," but it does not move forward or back; the position of the "clearest imagery" has now moved from x to y.

ASTIGMATIC LENS

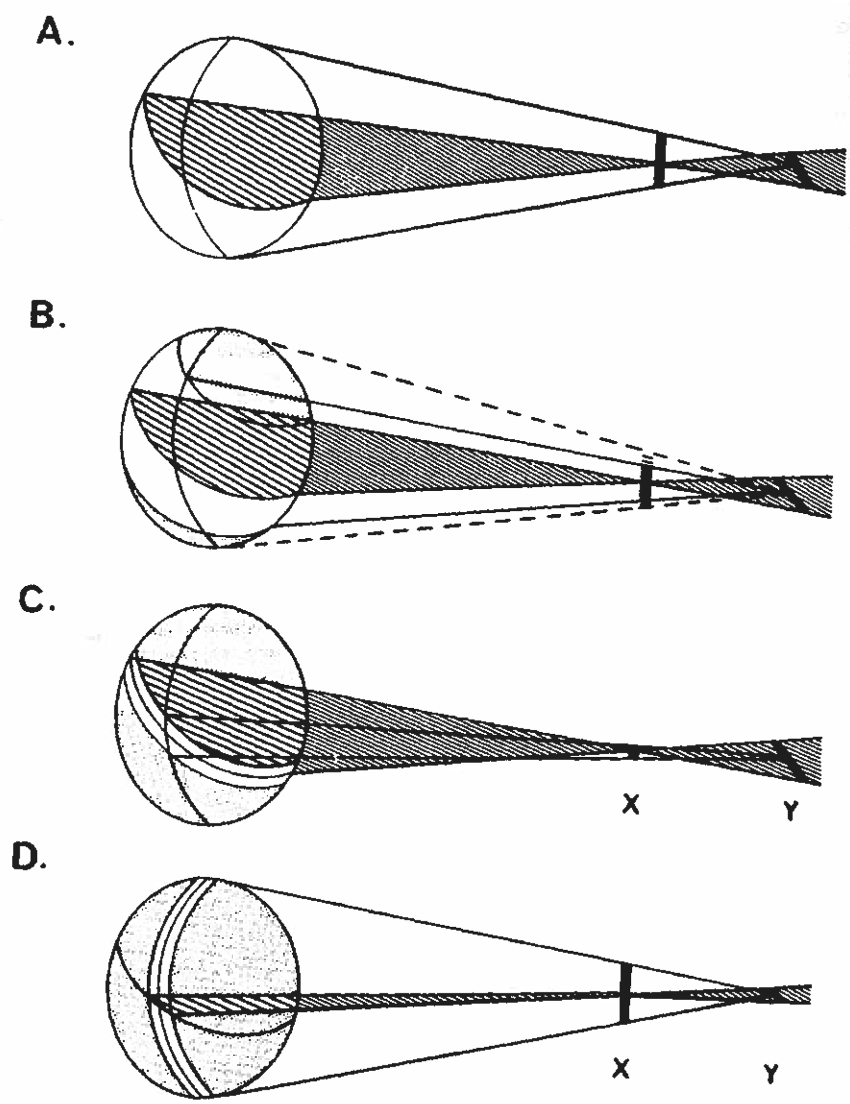


FIG. 3. *A*, the optics of an astigmatic lens forming two-line foci. *B*, the vertical aperture is narrowed slightly. This shortens the vertical focal line, without affecting the horizontal. *C*, the vertical aperture is narrowed to a horizontal slit; the vertical line is now point-like — still there is no effect on the horizontal line. The horizontal slit-like aperture is comparable to the stenopeic slit. *D*, with the slit vertical, the *horizontal* focal line becomes point-like; now the vertical focal line is unaffected.

We should be able now to surmise how to use the stenopeic slit clinically. It should be clearly understood, however, that the slit is helpful only as a *guide* to the subjective refractive error. Retinoscopic findings, if they can be obtained, are vastly superior to stenopeic approximations. On the other hand, if you have no old correction to go by and the retinoscopic reflex is of poor quality (secondary either to opaque membranes or lens capsule remnants, or affected by a pupillary opening too small to yield a recognizable reflex), the stenopeic slit can be very helpful.

Spherical lenses should be used by trial and error (by diopter or even two-diopter steps) to position the conoid somewhere in the "ball park" of the retina. Do not try to get the "best" acuity with spherical lenses, since in doing so you will probably bring the circle of least confusion on the retina; this will only complicate the cylindrical error determination to follow. When a reasonable acuity, say 20/200 or better, has been arrived at, bring up a stenopeic slit into the plane of the trial lens frame and rotate it to the angle that provides the best vision. If the patient claims that there is no "best" position, there are some obvious possibilities: (1) no astigmatism is present, (2) his astigmatic interval is equally mixed (equally spaced around the circle of least confusion which now is on his retina) or (3) he is physically incapable of recognizing any change in the image "quality" caused by the rotation of the slit. To test the possible existence of (2), throw in a + 1 or − 1 diopter sphere on top of whatever sphere you have in place, and try rotating again. If the patient still cannot see any difference, forget the whole thing and prescribe the best *sphere* determined by trial and error.

If, on the other hand, the patient is able to notice a difference in acuity with various positions of the slit (this is the most likely situation), place it in the position of best acuity and add plus or minus sphere on top of the slit to give the patient the sharpest possible vision. (You might need to refine the slit position slightly.) What you are now doing is "pinholing" the focal line positioned perpendicular to the slit opening, and moving that "pinholed" line onto the retina with sphere. (With the slit vertical, the horizontal line is pinholed as shown in Figure 3D). The *total* amount of sphere present (say, + 11 D to start with, plus 2 D more to clear up the target with the slit in whatever position you find — say at 90°) gives you the dioptric error of the focal line perpendicular to the slit (here, + 13 D for the horizontal line).

Next, rotate the slit exactly 90° to the first position. This blurs the existing retinal image and transfers the "pinholing" effect to the other focal line. You will now have to add *either* plus or minus sphere, whichever improves the acuity. (In using this method, you have no way of *knowing* which direction to go; you will have to try both, by trial and error, and determine whether the addition is plus or minus. Actually, that is the beauty of this technique. You "automatically" come out with the correct cylinder sign.) If you have had to add a plus sphere to clear the retinal image, the newly "pinholed" line must, before your lens addition, have been behind the retina, the plus sphere serving to pull it forward onto the retina. If you had to use minus sphere, the newly "pinholed" focal line was of necessity in front of the retina and your minus sphere pushed the line back onto it. It should be clear that whatever amount of sphere addition was necessary *that* is the amount of astigmatism present. If a plus sphere was necessary, then a corrective *plus* cylinder is required for a final corrective lens; if a minus sphere was necessary, then a corrective *minus* cylinder is required in the final prescription. In either case, the corrective axis is perpendicular to the slit in its second position; however, it is much easier to think of prescribing the corrective cylinder axis *parallel* to the *first* position of the slit. (Say you require − 3 D more sphere with the slit horizontal; the cylindrical prescription is − 3 × 90°. The *spherical* correction is that amount of sphere found to give the clearest vision with the slit in its initial position, + 13.00; final prescription, then, is + 13.00 − 3.00 × 90°.)

SUMMARY OF STEPS

1. Approximate sphere (without slit) to measurable, but not best, acuity.

2. Rotate slit to best angular position — "position 1."

3. Add plus or minus sphere to get the best acuity at slit-position

1. Refine angle slightly, if necessary: the *total* sphere in position 1 is the corrective sphere of the prescription.

4. Rotate slit 90° beyond position 1; this is "position 2."

5. Add plus or minus sphere to maximum acuity; the additional sphere is the cylindrical power which must be added to the corrective sphere of step 3. The cylinder is plus if the added sphere is plus; the cylinder is minus if the added sphere is minus. The corrective lens cylinder axis is parallel to the slit in position 1.

With the optics and the limitations of the technique described, you should be able to use the slit intelligently and "refract" patients who may have been "impossible" previously. At the very least, you will be able to recognize what a stenopeic slit looks like!
(Article *finis.)*

Another article, *"The Little Point That Isn't There,"* deals further with the effect of the pupil's size on visual acuity. It is reprinted in Appendix **C** .

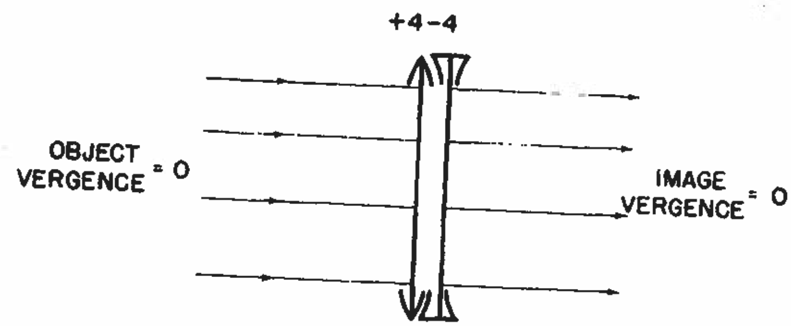
NEUTRALIZATION

There are a number of ways to skin the cat of ametropia correction: we have already considered one — far point correction. However, I would like to expound on another concept of correction to introduce a new subject — that of "neutralization."

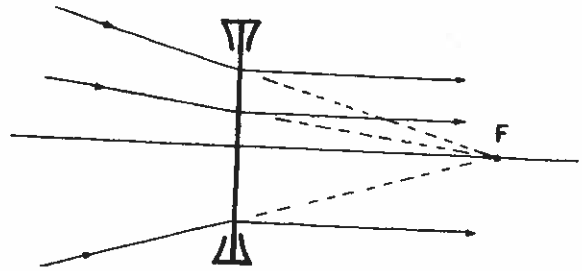
In all ametropias there is a mismatch of refractive power and axial length; since the latter cannot easily be tampered with, we must resign ourselves to dealing with the refractive power. What we would like to do is to make the eye of "proper" power to correspond to whatever its axial length is — to add dioptric power to an eye which is too weak (hyperopic) or reduce dioptric power (by adding minus lenses) to an overpowered eye (myopic).

If we place a minus lens in direct *contact* with a plus lens, the two powers add algebraically. When both are of identical powers but of opposite signs they "neutralize" each other and leave no resultant refractive power. We can do the same thing with an eye. However, we do not try to "neutralize" its entire refractive power, but only its refractive *error.* An eye which is myopic by 5 D is too strong by 5 D and this can be neutralized by a lens which is a — 5 D lens. This lens must, however, be in contact with the anterior surface of the eye. This same concept holds true for the hyperopic, "weak" eye in which we must supply dioptric power in the form of a plus lens in contact with the cornea. Now why do I stress *contact* between the corrective lens and the anterior refractive surface?

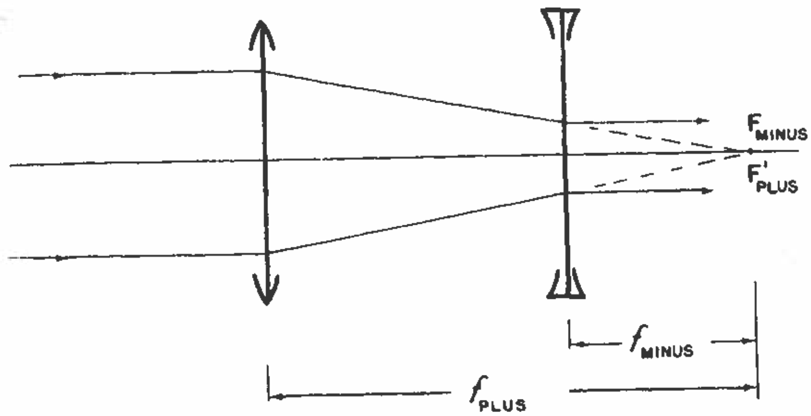
Neutralization involves the "balancing out" of any dioptric power between two lenses leaving an *afocal* system; that is, if the incoming vergence is zero, the outgoing (image) rays for the system of lenses will also have zero vergence. When the lenses are in contact, a minus lens is *exactly* neutralized by a plus lens of equal power.



When we separate the two lenses, and we still want to neutralize the − 4 D lens, the plus lens will have to be of less power dioptrically to produce an afocal system. Let's see how much plus is required of the "neutralizing lens". For a − 4 D lens to produce zero *image* vergence, we must have light rays in its object space directed to its primary focal point F. But F of a minus lens is on the *right* of the lens. Thus, light rays in the object space must be converging toward F to enable them to leave in parallel bundles.



To create that convergent beam, we require a plus lens. We know that this lens can take parallel entering rays and converge them towards *its* secondary focal point F'. So, if this F' is made to coincide with F of the minus lens, our conditions for a "neutralized", afocal system will be satisfied — the original object vergence is zero and the final image vergence is also zero.



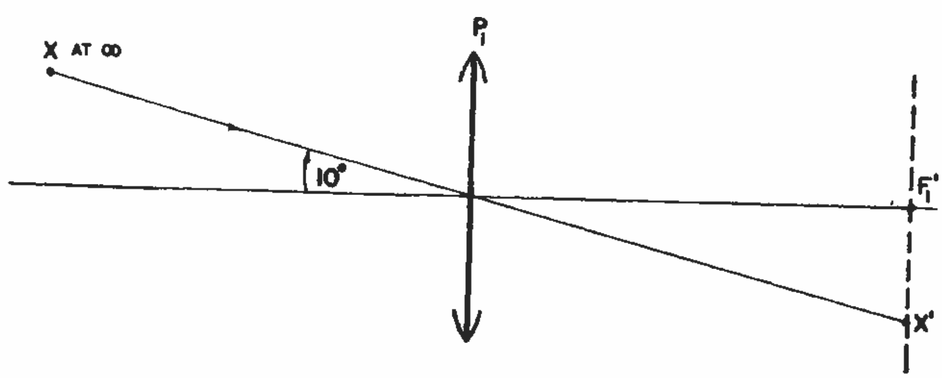
It should now be clear that we can neutralize a minus lens with a plus lens. When the lenses are in contact, the secondary focal length *f'* of the plus lens will be the same length as the primary focal length *f* of the minus lens — thus, both are of the same power. However, when there is any space between the two lenses and we desire a coincidence of the focal points to occur — the condition for neutralization — *f'* of the plus lens must be the longer focal length. The greater the distance between the lenses, the longer *f'* of the plus lens would have to be and, therefore, the *weaker* its plus power. So now we've shown that for the neutralization to occur, the plus lens power will always be equal to (when in contact) or *less than* (with separation) the power of the minus lens.

Though we have been talking about "hand-held" plus and minus lenses, the identical principle holds for "eye error" and "corrective lenses", that is, when we try to "neutralize" any refractive error. A hypermetropic eye has too little power for its axial length and can be considered as having a small extra *minus* lens "built" into the eye. It is corrected (neutralized) by a plus corrective lens which must be greater in power the closer it is to the "minus" lens within the eye. The *"corrective"* lens would necessarily have to decrease in plus power if it were placed further from the eye error. (This is just another way of looking at the identical point which I made when dealing with far-points and corrective lenses!)

So in two different ways I have emphasized that the "proper" corrective lens power can vary yet still be "corrective". Until now, I have avoided the question of what the *patient* notices when one changes the corrective lens power. Let us examine what happens during such power manipulation by closely scrutinizing our "neutralized" afocal system (also called a Galilean Optical System).

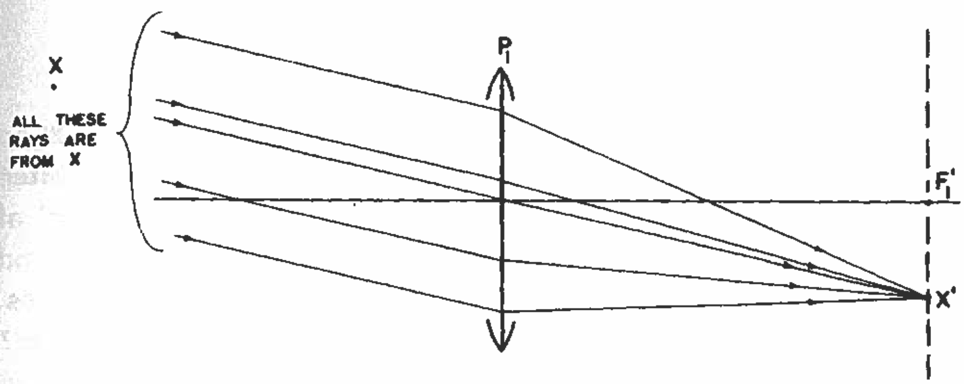
Galilean Optical System

Our previous figure showed that light rays with zero vergence from an object point at infinity also left the system with an image vergence of zero. Though this is true for all object points at infinity, that diagram shows only the rays from that particular point at infinity which is *on the axis*. What about a point at infinity which is off the axis — say one 10° above the horizon?

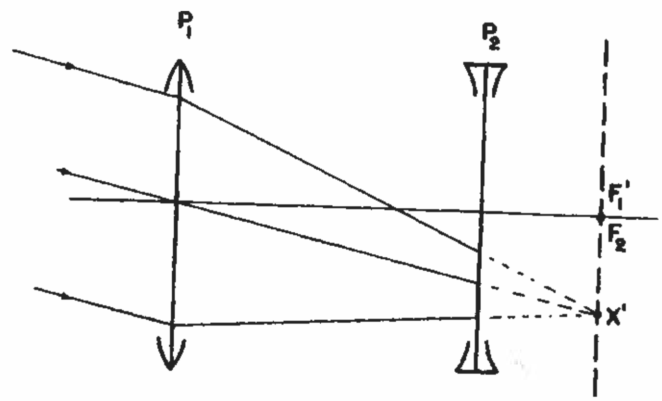


Our first lens P_1 would image this point in its secondary focal plane. Since the object is at infinity but off the axis, the image point is determined by the ray from X through the P_1 nodal point.

We can draw a few more rays from X.

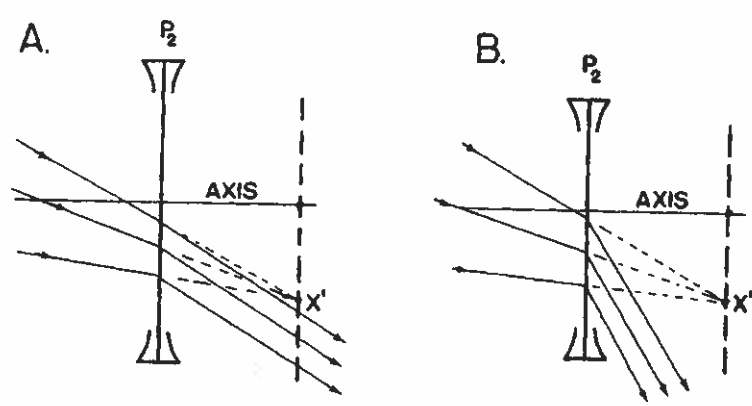


Since X is at infinity, those rays which reach lens P_1 will all be parallel and all will have the same 10° inclination to the axis, and after being refracted by P_1, all rays will be bent toward the image point X' in the secondary focal plane (as we saw way back in the early chapters). However, in our Galilean neutralized system, these rays in the image space of P_1 cannot actually reach X'; they are intercepted first by our minus lens P_2, which is situated in front of the secondary focal plane of P_1.

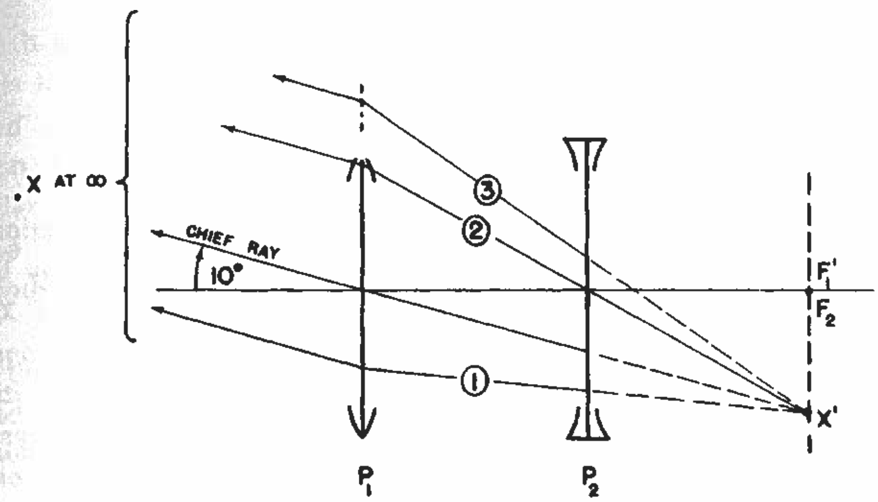


P_2 "sees" X' as an "object" point and images it — the question is where? We know that X' is in the *object* space of P_2 and is located in the primary focal plane of P_2. (Remember our coincidence of F' of P_1 and F of P_2?) The image of X' by P_2 then must be at infinity, and all the rays originally heading for X' must leave P_2 in parallel bundles.

O.K., so far, but how do we know at what angle these rays leave? They *could* be bent in any direction (two possible ones are shown in A and B below) and still they could remain in parallel bundles.

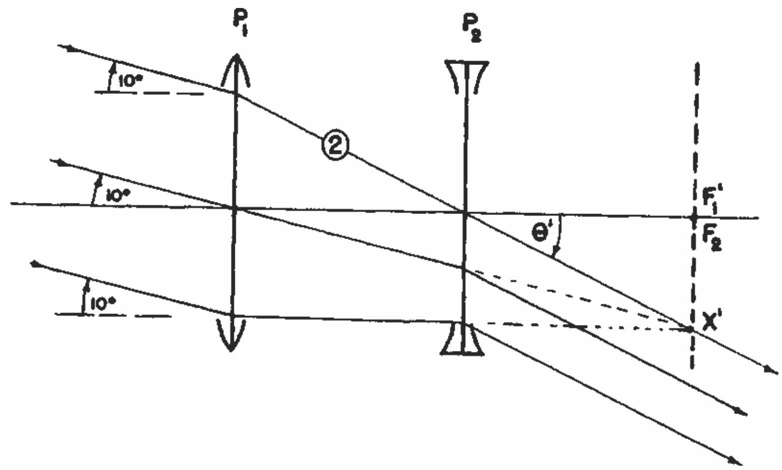


Can you see how we can find the final direction imparted to these parallel bundles of rays? You should. We know that *all* rays from the *original* object point X had to be imaged by P_1 at X′. Therefore, we could trace backwards *any* ray originally directed to X′ (such as 1, 2 and 3 in the figure below) and they would all have to be parallel to all those original *object* rays with an inclination to the axis of 10° — the same as our "key" ray through the P_1 nodal point.



(In order to draw ray 3 back from X′ to P_1, we had to artificially extend the lens plane P_1 upward so that 3 would "intersect" it and change its direction — that ray, in the object space of lens P_1, also makes an angle of 10° with the axis.)

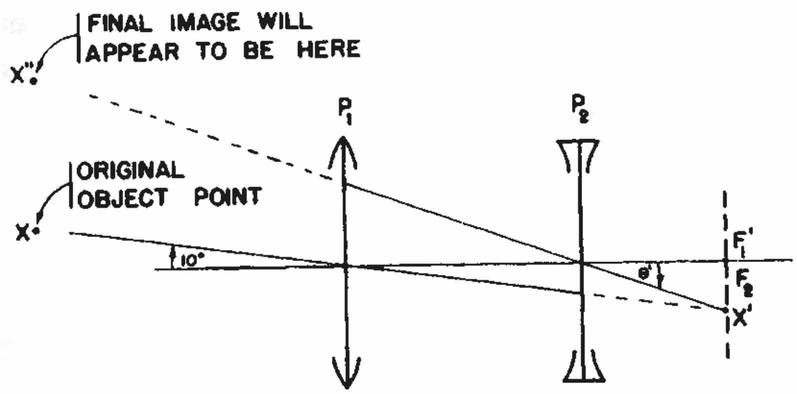
In any case, we know that there will be one ray which passes back from X′ through the nodal point of P_2 (ray 2) and in doing so will not be deviated. AHA! That ray 2 will be the one that determines the *final* direction of *all* image rays from P_2 for which the object was X′. That angular direction is shown in the diagram below as θ'.



In summary, ray 2, originally from object X, hits P_1 at an inclination of 10° to the axis. P_1 refracts that ray toward X'. Though this ray is intercepted by P_2 (and will thus participate in the *final* image of X' which will be at infinity), it is *not* deviated further since it is the *one* ray which passes through the P_2 nodal point. Since we now know *its* direction, we also know the final direction of all the final image rays.

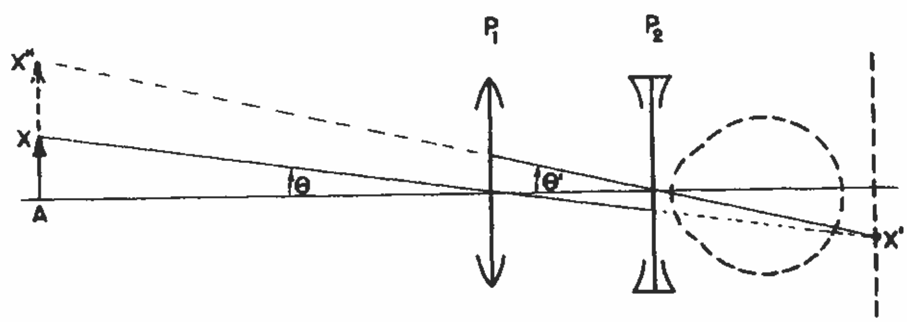
(By the way, our last diagram shows that the final image rays look as if they are "packed" closer together than their corresponding object rays. Many are led to the *erroneous* conclusion that image *mini*fication must have occurred to cause this "packing". You should not be led astray. All those image rays stem from the *same point* as the object rays. Therefore, their apparent density cannot signify magnification or minification. It is only their angular or directional *change* that matters in the determination of magnification.)

We can simplify the above diagram and show only *one* object ray inclined at 10° and *one* (the undeviated) image ray inclined at θ' which is obviously *greater* than 10°. The final image of our original object point X will be at infinity (since there are parallel light bundles in the final image space), but that image will appear to have a *greater* inclination to the axis than the 10° inclination of our original object point.



It should be obvious what an eye will see when placed behind P_2 if it looks at the final image point X'' through this afocal, "neutralized" lens system. Since X'' is at a greater angle above the horizon than X, the entire object must appear larger — magnified in its angular size. This system of lenses is a Galilean or terrestrial telescope. The image seen will be virtual, upright and *magnified* whenever the *minus* lens is closer to the eye.

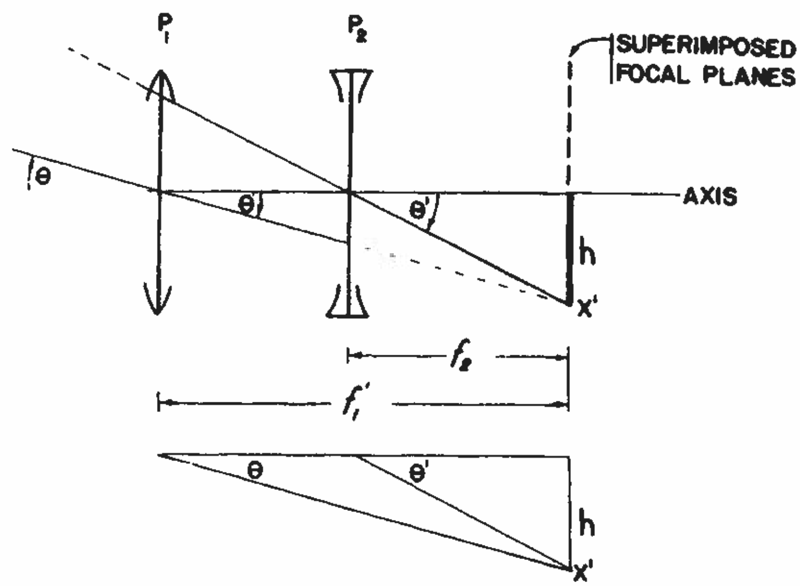
You should draw a diagram for yourself demonstrating that if it is the plus lens that is nearer your eye, the image seen will be minified. (Even in this latter, afocal *mini*fication system the plus lens power will always turn out to be less than the minus lens power.)



Without the Galilean magnification system shown above, the eye would see object AX; *with* this telescope, it sees image AX'' which would occupy a greater extent of its retinal surface area. The actual magnification, of course, depends on the relationship of θ (the angular size of the object) and θ' (the angular size of the image). This angular magnification M is $\frac{\theta'}{\theta}$.

If θ' is two times as large as θ, the angular magnification would be 2 or 2X — signifying a *100%* increase in the apparent size.

Now let's look and see how we can denote the angular magnification of a telescope if we know the lens *powers* we are "neutralizing". Again to simplify, only one object ray and another image ray are drawn to X'. Image X' is located a distance *h* from the axis:



$$\tan \theta = \frac{h}{f_1'}$$

$$\tan \theta' = \frac{h}{f_2}$$

Since for small angles $\tan \theta = \theta$ and $\tan \theta' = \theta'$,

$$\theta = \frac{h}{f_1'} \text{ and } \theta' = \frac{h}{f_2}$$

$$M = \frac{\theta'}{\theta} = \frac{\frac{h}{f_2}}{\frac{h}{f_1'}} = \frac{f_1'}{f_2}$$

In contrast to what you will find later when you study the *astronomical* telescopic magnifying system, in this Galilean system both the object and its images are upright so both θ and θ' are measured in the same (here, clockwise) direction; therefore they must be of *like* sign. But recall that to attain this, we had to make P_2 a *minus* lens (with its minus f_2), so our *general* expression will have to be as follows: $M = \frac{\theta'}{\theta} = -\frac{f_1'}{f_2} = -\frac{P_2}{P_1}$. This is the basic magnification relationship between *any* two lenses which "neutralize" each other. Now if P_2 happens to be a minus lens as it is here, M will turn out to be plus, signifying an upright image — *that* was our originally agreed upon convention.

Magnification Following Optical Correction

At the beginning of this section, we stated we could consider the hyperopic eye as being "too minus", with a built-in minus lens inside — this "lens" is P_2. This "error" could be "corrected" by any lens P_1 outside the eye; the greater its distance from the eye (that is, the greater the "vertex" distance), the less plus is required in the corrective lens. Finally now, we can see the consequences of varying the position and power of the corrective lens. The further from the eye the corrective lens is, the less its plus dioptric power; but, the *greater* is the magnifying power it presents to the eye! So, as P_1 (the "corrective" lens power) decreases, the magnification $(\frac{P_2}{P_1})$ increases.

If P_1 is a plus corrective *contact* lens, the magnification $\frac{P_2}{P_1}$ induced by that corrective lens for *that* eye will be minimal — that is, less than that created by a corrective lens in *any* other position (aside from an *intra*ocular* lens!). But keep in mind that even with a contact lens, there will still be *some* magnification present. This is because the eye "error" is not *at* the cornea but somewhere within the eye; that is, some physical separation will still exist between the "error" and its corrective, "neutralizing" lens.

* Such as a Binkhorst lens for an aphakic eye.

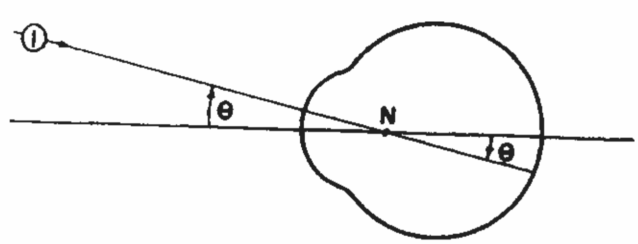
CLINICAL POINT:

These optical fundamentals provide the basis for a handy gimmick for aphakes. We have already seen that an aphake of 12 D (with a refractive *error* of − 12 D measured at the cornea) can be "corrected" by a + 3 D sphere if it is held about 25 cm from the cornea. (You should be able to diagram *exactly* where it must be held!) Now it is clear that such a "correction" will provide him with $-(\frac{-12}{3})$ or 4X magnification (that is a 300% enlargement) of the retinal image of some object! This simple do-it-yourself telescope with a + 3 D sphere is especially useful to that aphake who has a reduced visual acuity.

* * *

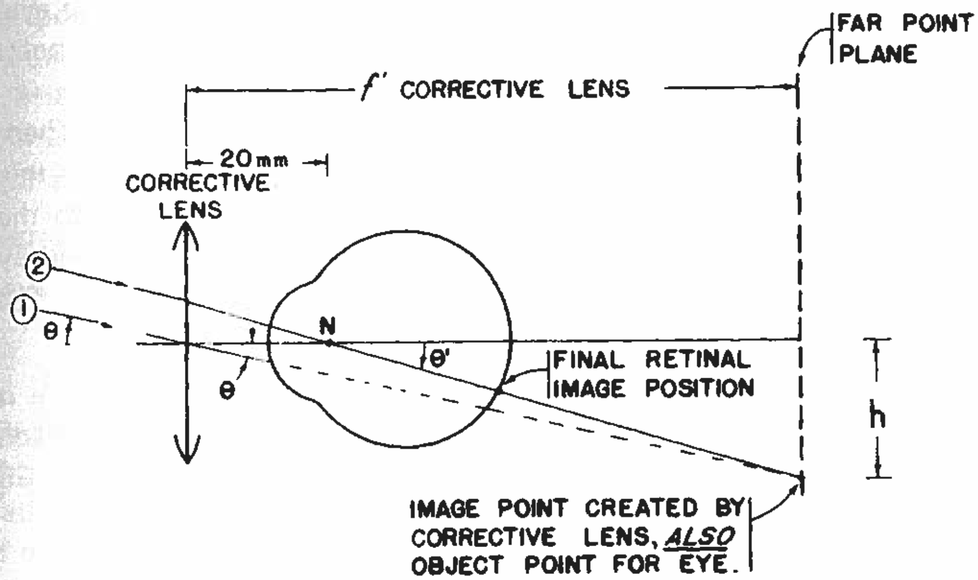
Magnification is nice when it's necessary but can be a handicap too. Take for example the unilateral aphake (the other eye being emmetropic) — here, even a contact lens correction will create a 7% image magnification to impede a sensory fusion with the image given by the emmetropic eye. I have always been amazed that most individuals corrected this way do *not* have difficulty!

Typically, the average aphakic eye has its corrective lens worn in the spectacle lens plane, 15 mm from the cornea. This affords a magnification factor of about 125% (usually written as "25% magnification and implying an image which appears 25% larger than the object itself). Oops! That's putting the cart before the horse. You yourself should be able to diagram the comparison between the magnified retinal image seen by a corrected aphakic eye with that by an emmetropic eye; but anyway, let's go through it together. First the emmetropic eye:



The angular size of the object is given by ray 1 which goes from the tip of the object to the eye's nodal point. This ray subtends angle θ with the axis. Since ray 1 is undeviated (through N), the angular size of the retinal image is also θ. (Remember that the nodal point ray in *any* eye provides a quick reference to the angular size of the retinal image.)

Now to the corrected aphakic eye:



Follow this reasoning closely. In contrast to the emmetropic eye, the aphakic eye requires a corrective lens which here is positioned about 15 mm from the cornea or about 20 mm from the nodal point of the eye. Of course, the secondary focal plane of any "corrective" lens will coincide with the aphakic eye's far point plane. This corrective lens will image every object point from infinity somewhere in its focal plane. Since that image point also happens to be in the eye's far point plane, the aphakic eye will see it clearly.

The original object point (just as in the emmetropic eye) subtends object angle θ at the lens. The corresponding image point is located in the secondary focal plane of the lens by ray 1 through the lens' axial (nodal) point. That is, if this actual ray were *not* intercepted by the eye, it would proceed undeviated and intersect the secondary focal plane (and far point plane) at a distance h from the axis. This intersection, then, is where the corrective lens images the original object point.

Now get this; once this image is formed by the corrective lens, the eye couldn't care less about that lens; it is only this image point which assumes importance as it becomes the *object* point for the eye. This object point subtends an angle of θ' at the nodal point of the eye. With reference to the eye then, this "object" point (in the far point plane) and the final retinal image point must both lie along the undeviated ray 2 which passes through the eye's nodal point. (There should be no question about that.) If you now want to see where this ray 2 *originally* came from, you must trace its path back through the corrective lens; you will then find that it must have been refracted by that lens so that in the *lens'* object space it was parallel to ray 1 (as shown in the diagram above).

Since θ' is angular size of the *retinal* image, we can compare it to θ, the angular size of the original object and arrive at an expression for the angular magnification given by an aphakic spectacle lens. Go back to the last diagram for a little geometry:

$$\tan \theta = \frac{h}{f'}; \ \tan \theta' = \frac{h}{f' - 20 \text{ mm}}$$

Again with small angles, $\tan \theta = \theta$ and $\tan \theta' = \theta'$

$$\frac{\theta'}{\theta} = \frac{f'}{f' - 2.0 \text{ cm}}$$

Since this sample aphakic corrective lens can be considered to have a focal length of about 10 cm, substitute 10 cm for f'.

Therefore, $\frac{\theta'}{\theta} = \frac{10}{8} = 1.25$

or $\theta' = 125\% \ \theta$

Thus, θ' for an aphakic eye is 25% larger than θ, which is the image size for an emmetropic eye.*

Study the next diagrams for a moment. (Remember, the far point plane stays in the same fixed position relative to the eye and is independent of where *any* "corrective" lens is placed.)

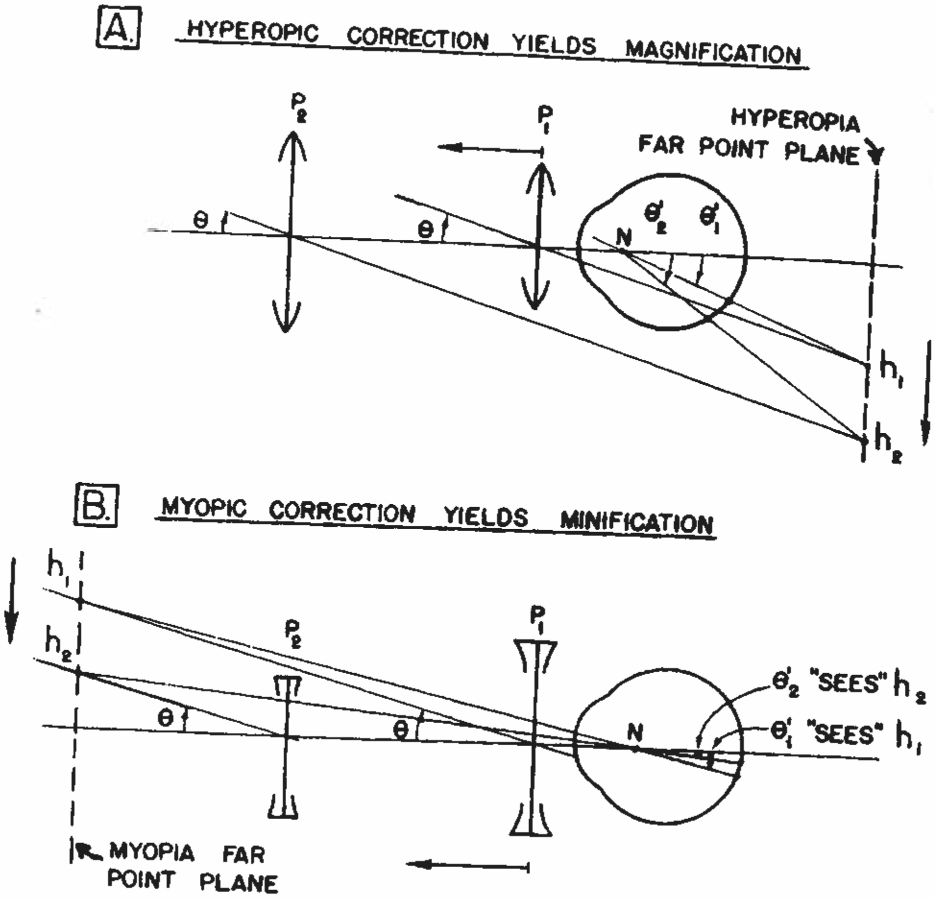
Figure A (hyperopic eye): for a given angular object size θ, in the far point plane there will be a corresponding image size h_1 created by that corrective lens. As the corrective lens P_1 moves *away* from the eye towards P_2, h_1 must move towards h_2. So, the size of this image will get longer and, as it does, the angular size θ' of the *retinal* image will also necessarily *increase*. Hence, as the vertex distance of a plus corrective lens increases, the Magnification $\frac{\theta'}{\theta}$ increases too, as we have cried so often.

Figure B (myopic eye): An object of angular size θ will be imaged by lens P_1 in the myopic eye's far point plane at h_1. So for the eye, the original object will appear to be at h_1. The angular size of the retinal image of h_1 as subtended at the eye's nodal point will be θ_1'. If now you move the corrective lens toward P_2, h_1 will move toward h_2 and the angular size θ_2' of the retinal image will shrink. So here, the Magnification $\frac{\theta'}{\theta}$ *decreases* as the vertex distance of a minus corrective lens *increases*.

In the diagrams below, then, picture that in both hyperopia and myopia, as P_1 moves away from the eye towards P_2, h_1 (in the far point plane) will move toward h_2 and the retinal image size θ_1' shifts towards θ_2'.

---

* Thus, any corrected aphakic eye should be able to read smaller-sized print (and hence demonstrate a "better acuity") on a Snellen chart than would an emmetrope.



In both A and B above,
as lens P_1 moves towards P_2, h_1 moves towards h_2.

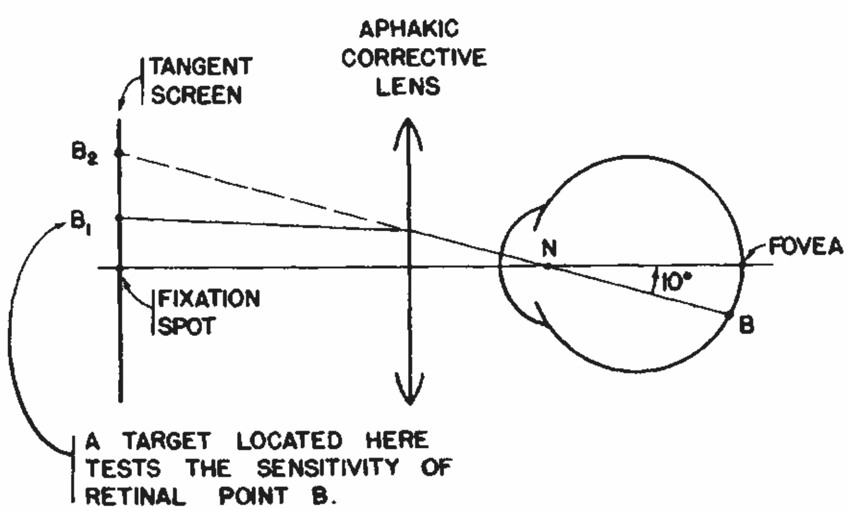
The corrected myope has magnification working in reverse for him, as if he looked through the telescope the wrong way. This introduces minification — a *reverse* Galilean effect. The further away from his eye he has his corrective lens mounted, the greater the *minus* power must be, and the more *mini*fication he obtains. His most magnified view of the world will come with a contact lens. No wonder then, the great affinity that myopes have for contact lenses. A 10 D myope will obtain about a 15% increase in retinal image size with a contact lens compared to that afforded by a corrective spectacle lens.

The magnification and minification effects obtained by varying the vertex distances account for many everyday-observed phenomena. Not only will a myope see spatial objects as minified, but, if another observer looks at this myope's fundus through an ophthalmoscope, he will see it *magnified* over that visible in emmetropes. This is because he must use a minus lens in his ophthalmoscope head to compensate for the "built-in" plus error of the myope's eye; and since there will always be a "vertex" separation, he will obtain the same optical effect as when looking through a small Galilean telescope. The *minus* lens is closer to the examiner's eye, and therefore, the image seen is magnified. Vice versa for the image of a hyperopic fundus.

CLINICAL POINT:

A spectacle-corrected aphake has a large retinal image. This means that the image of a given area of space now falls on a greater area of the retina than it did prior to the aphakia. A spot in that retina (say 10° nasal to the macula as subtended at the eye's nodal point) will, through the aphakic corrective lens, project outward onto a tangent screen to a position that is *closer* to the fixation point than would the comparable retinal spot of an emmetropic eye.

Look at a sketch:



The fovea is aligned with the fixation spot on a tangent screen. In an emmetropic eye, retinal point B, 10° below the fovea, would normally project out onto the screen to point B_2. Because of the presence of the aphakic corrective lens, B projects instead to B_1. Now look at this same situation from the point of view of an examiner at a tangent screen: If a given sized test object is to be just barely detected by (at the sensitivity threshold of) retinal area B, it would have to be placed at B_1, which is closer to fixation than B_2. If it were placed at B_2, its corresponding image on the retina would necessarily be *below* B and therefore would become a subthreshold stimulus for that retinal area.

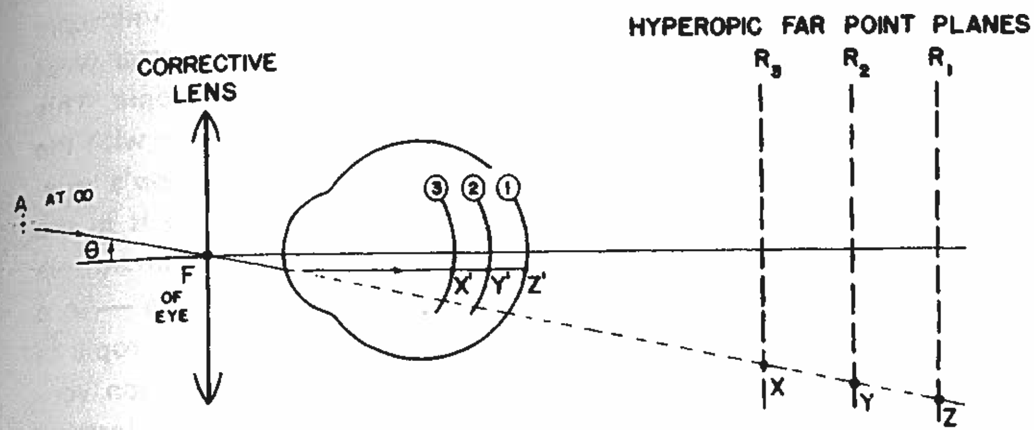
This same reasoning can also be applied to the corrected aphake's visual field isopter lines as mapped on a tangent screen; they will be closer to the fixation point than in the non-aphake. Also, the blind spot will be smaller and closer to fixation than will that of a patient who is not aphakic. (Obviously, the reverse is true for a high myope.) You must be aware of this when you plot the field and attempt to search for the blind spot in highly ametropic, "corrected" patients.

* * *

As stated, the magnification effects described so far are those due to vertex distance or the *separation* between elements which are being "neutralized". There *is* a particular vertex distance which has unique properties. That distance is one which places the corrective lens in the anterior focal plane of the eye, roughly 15 mm from the cornea.

Knapp's Rule

Let us construct the retinal image in an eye which is *hypermetropic*, but so by virtue of its being too short (that is, an *axial* error). We will consider simultaneously *three* such eyes, with the retinas in three separate positions. The anterior segment will be identical for all three eyes since the "error" is in the length, not in the power.



KNAPP'S RULE

Each eye will have a corresponding far point plane. Eye 1 corresponds to R_1, etc. Since we have assumed that all of these eyes have the same refractive power, F (the anterior focal point) is in the same position for all these eyes.

If any of these three eyes is to see point A (at infinity) clearly, a corrective lens is necessary to image it in the corresponding far point plane. Let us place the appropriate corrective lens in the anterior focal plane of the eye so that the lens' own nodal point coincides with F of the eye.

Draw a ray from object point A that crosses the axis at the lens nodal point. For all three corrective lenses, that ray will continue on undeviated. Extended beyond the eye, this ray indicates the position in each of the corresponding far point planes where the image of A will be — at X, Y and Z, respectively. (The actual sizes of the images in each of these planes will differ, the largest image being present in the most remote plane). However, since this same ray passes through F of the eye, it will, after refraction by the eye, emerge (in the vitreous) parallel to the axis. You can see that the position of the retina (which would vary with the degree of axial ametropia) will not influence the *absolute size* of the retinal image of A; images X' Y' and Z' are all the same distance from their respective maculas.

(The foregoing analysis also covers myopic correction with the myopic far point plane in front of the axially myopic eye. This will not be diagrammed here.)

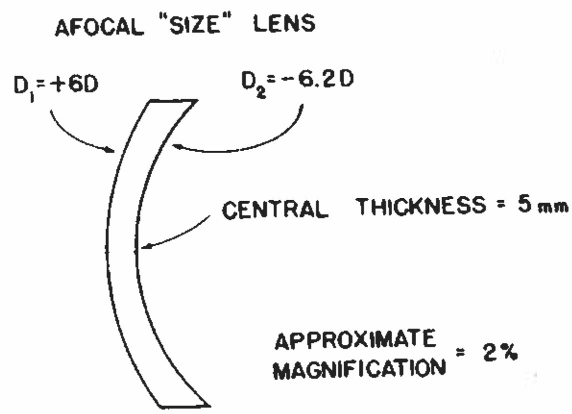
Thus, a proper corrective lens located at F in any *axial* ametropia will produce retinal images of the same actual size, no matter what the degree or direction (myopia or hyperopia) of the ametropia. This retinal image size will be the same as that given by any eye with the identical *refractive* power, emmetropic or not. This is Knapp's rule. Know what it is, but realize the obvious difficulty in using it in the clinical situation; you can never know whether an existing ametropia is axial or not (though sometimes, you may have a fair idea — in a monocular high myope, for example, it is likely that the ametropia is axial). In general then, I would like to state my heretical opinion very clearly. For practical purposes, it doesn't make any *real* difference whether an ametropia is axial or refractive; so, Knapp's rule has no *useful* application though it theoretically seems like it should!

Aniseikonia

When a patient *perceives* a difference in the image sizes seen by his two eyes, no matter what the cause, the defect is labeled *aniseikonia.* The signs and symptoms of this malady are well discussed in other clinical texts. Suffice it to say, size *differences* of less than 5% can induce symptoms. Clinically the size difference perceived by a patient can be measured on an instrument called an *eikonometer.*

Lenses which compensate for this difference can be ground and are called "size" lenses. These are really miniature Galilean magnifiers (or minifiers) with a very small (in millimeters) separation between elements — this separation, however, is one factor which helps create the few percent change in image size which may be desired. (In the "size" lens, the space separating the front and rear dioptric elements is not usually filled with air but is composed of the lens material itself.)

The second factor helping to provide the magnification given by a "size" lens stems from its front and back surface powers which can be varied to change the "shape" of the lens. Thus, even when *no* "corrective" dioptric power is necessary, a simple "size lens" can be made. One such lens is diagrammed below.



To give you some idea of the range of magnifications allowed by such a lens, I have constructed the small table below. From this table you can determine that a lens of 5 mm thickness and with a front surface power of + 6 D (the rear surface power would have to be about − 6.2 D for an *afocal* lens), would yield about 2% magnification.

TABLE IV

MAGNIFICATION IN %

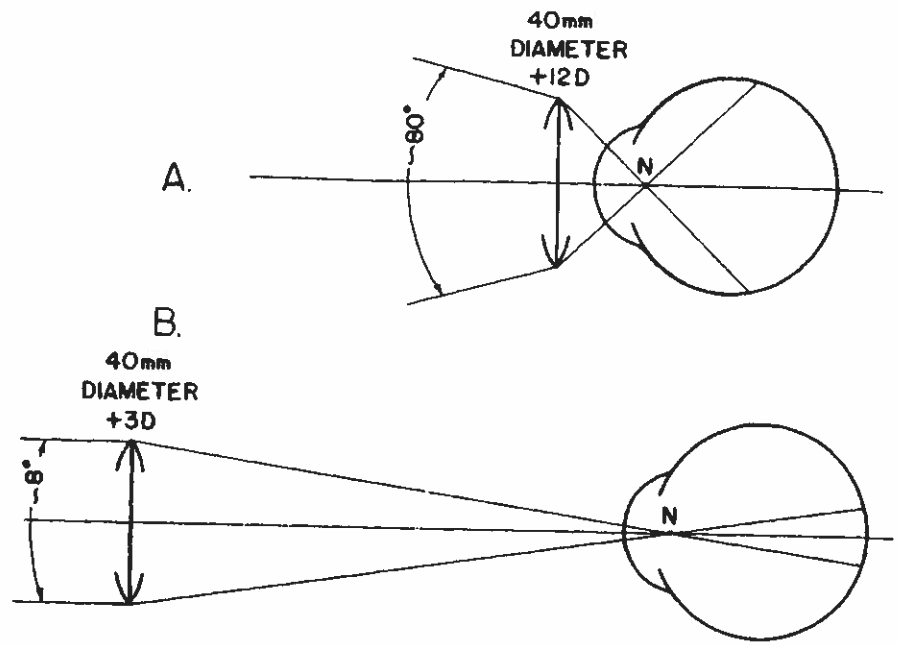
Lens Thickness (mm)	D_1 (Front curve in diopters) + 3 D	+ 6 D	+ 9 D
1	0.20	0.40	0.60
3	0.60	1.20	1.80
5	1.00	2.01	3.00

(*For the mathematical buff only* — The M in % $= D_1 \times \frac{t}{n}$

D_1 = diopters of front surface power
t = thickness in *cm*
n = index of refraction of the glass)

This table shows us that the *shape* of a lens (as governed by the steepness of its front surface) and its *thickness* influence its magnifying power, just as does the *vertex* distance between the lens and the eye; the latter, however, is more significant in everyday ophthalmic optics. The shape and thickness effects are simply a manifestation of "thick-lens" optics on which I do not wish to elaborate further.

We already mentioned the 4 X magnification an aphake might obtain with a + 3 D lens held about 25 cm from his cornea; however, he must give up something to obtain that magnification with this "corrective lens" and that something is "field of view".



FIELD OF VIEW

As can be seen from the above diagrams, when the corrective lens is in the spectacle lens plane, the field of view (that area of space that is visible through the lens) is much greater than when the lens is situated at a further distance. In the first instance, the field of view is about 80°, whereas in the situation which provides a magnification of 4 X, the field of view shrinks to about 8°.

Our study of "neutralization" of lenses and Galilean optics has led us through a consideration of the retinal image size changes occurring in the correction of myopia and hyperopia. Astigmatic correction with the lens situated in the spectacle lens plane presents the same type of problems; the presence of a vertex distance here also leads to magnification or minification. The difficulty is that this size change is not even — there is a differential, meridional magnification which occurs with the size extremes being in the primary meridians.

The *meridian* with more plus *corrective* lens power creates a greater amount of magnification than does the lesser powered meridian. To visualize this, recall the cross-section of a Galilean system composed of a *plus* corrective lens and a *minus* eye "error"; say the plane of that section was taken in the lens meridian of *greatest* plus power which must also be the meridian of greatest minus error. You should see that this arrangement will yield a *magnified* retinal image in this meridian which must be larger than that in *any* other meridian of this same eye. So, if the corrective lens power in the *vertical* meridian happens to be greater in plus than any of the other lens meridians, there will be *vertical* elongation of the image seen by the patient.

Take another example of a corrective lens of plano $+ 4 \times 90$. Here the maximum corrective plus power is in the 180° *meridian.* Therefore, the horizontal dimensions of the image will be elongated. If the object is a square, the retinal image will look like a horizontal rectangle.

I will not explain this further here but you can prove it to yourself. Place a $- 5 \times 90$ cylinder close to your eye to simulate an astigmatic eye's "error". "Correct" this error with a $+ 4.00 \times 90$ held at a "vertex distance" of about 5 cm. You should see that the *horizontal* dimension of any object (say a square) is elongated. The greater the distance the "corrective" lens is held, the more this horizontal magnification is.

Whenever the primary meridians are at 90° and 180°, rectilinear square objects may appear elongated sideways or up and down, depending on the corresponding powers. However, this type of apparent distortion is usually *not* very bothersome to a patient. Even when the corrective lenses have oblique axes (which may cause right angled corners to look somewhat acute or obtuse) patients become very tolerant within a short adapting period of a few days or weeks.

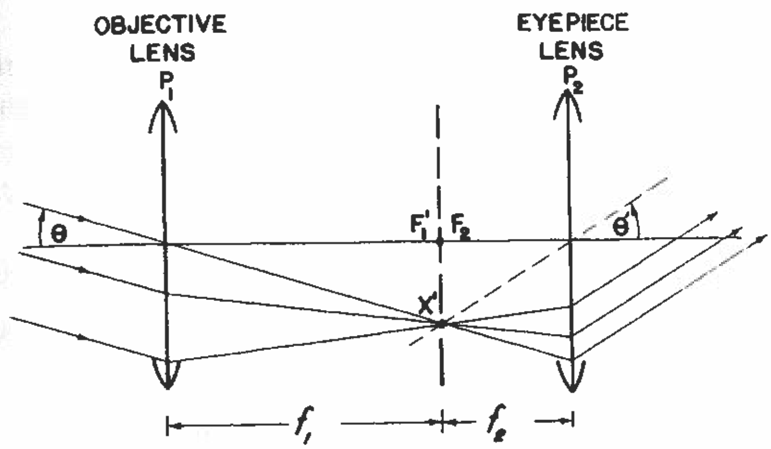
CLINICAL POINT:

A problem may arise, however, when the primary meridians are oblique *and* different in the two eyes, thereby giving a different "tilt" to an image line for each eye; this would necessitate a cyclofusional eye movement to keep from seeing diplopically. You can help reduce this "tilting" type of distortion by *under*correcting the cylindrical power somewhat; this technique is probably better than attempting to "straighten" the distortion by rotating the corrective axis toward the horizontal or vertical meridians (a common practice), since the latter method tends to introduce not only a resultant cylindrical error but also adds a spherical one, as we have seen. (Remember what happens when you combine unlike cylinders off-axis to one another?) However, both methods do work.

Astronomical Telescope

The Galilean telescope and the principles of its operation are closely related to ophthalmic optics and the correction of refractive error; however, this instrument is not the only *afocal* system providing angular magnification. There is another, extremely useful one called the "astronomical telescope" which produces an *inverted,* though magnified image; otherwise, the optics are very similar to the Galilean system's.

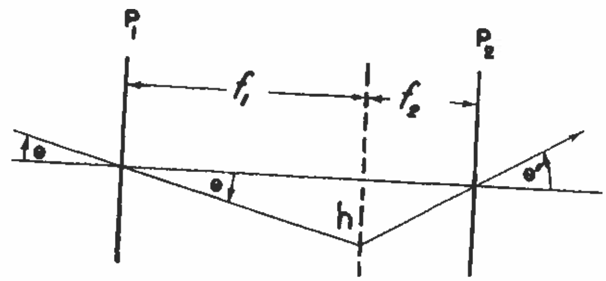
ASTRONOMICAL TELESCOPE



The front (*objective*) plus lens P_1 of this magnification device creates a real image in its secondary focal plane (F_1') of an object at infinity, just as it did in the Galilean system. But here the second lens P_2 (called the *eyepiece* lens) is *plus*; it is placed so that *its* primary focal plane F_2 coincides with the F_1' plane of the objective lens. The image point X' formed by P_1 will now act as an object point for P_2. Since X' is in the primary focal plane of P_2, its final image will be at infinity. As in the Galilean telescope, the direction (angle θ') that the final parallel bundles of light make with the axis is given by a line drawn from X' through the nodal point of P_2 (which is at its axial vertex). At the nodal point of P_1, θ is the angular size of the object.

The angular magnification power of this telescope is $\frac{\theta'}{\theta}$; but as can be seen from the diagram θ' and θ are in the opposite directions; the object denoted by angle θ is above the axis, while the image angle θ' points to the corresponding image X' located *below* the axis — this is in contrast to the Galilean system where both the object and image were in the same direction. So, $M = -\frac{\theta'}{\theta}$.

To simplify the above diagram (still considering small angles) look at the figure below:



$$\theta = \frac{h}{f_1}$$

$$\theta' = \frac{h}{f_2}$$

Since θ' is counterclockwise here,

$$M = -\frac{\theta'}{\theta} = -\frac{f_1}{f_2} = -\frac{P_2}{P_1}$$

Notice this is the same expression as that given already for the power of the Galilean telescope! Thus, the Magnification of *any* telescope is simply given by the relationship $-\dfrac{\text{power of the eyepiece}}{\text{power of the objective}}$.

From this, it should be clear that the *same* magnification power can be provided by *any* pair of lenses that yield a constant ratio of powers; a 3 X telescope can be made up of a P_2 of + 9 D and a P_1 of + 3 D, or a P_2 of + 21 and P_1 of + 7. (The distance between the lenses will necessarily be shorter in the latter pair.)

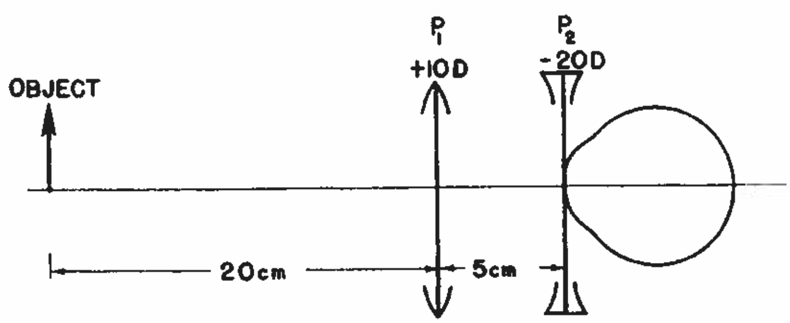
The astronomical telescope principle is utilized clinically in the indirect ophthalmoscope. Also, this same principle (modified somewhat to allow the real object to be very close to P_1) is the basis for the standard light microscope.

The Effect of a Telescopic Magnifier on Accommodation

Both the astronomical and Galilean telescopes are afocal systems, meaning light from a distant object emerges from the instrument in parallel bundles. Since this image vergence is zero, there is no accommodative demand for an eye looking at infinity through such a telescope. What happens to the image seen by an eye if the *object* approaches a Galilean system?

PROBLEM:

Assume an object is located 20 cm from the front (objective) lens of a 2 X Galilean telescope where $P_1 = +10$ D, $P_2 = -20$ D, and the separation is 5 cm. What is the accommodation required to see the object through this telescope compared with the accommodation demanded by the same object located at the same distance *from the eye* (25 cm) without any telescope?



ANSWER:

The object vergence at P_1 is -5 D.

$$U_1 + P_1 = V_1$$
$$-5 + 10 = +5$$

The image will be 20 cm to the right of P_1.

Since P_2 is located 5 cm to the right of P_1, the object *distance* for P_2 is 15 cm to the right (convergent).

$$U_2 = +\frac{1}{.15} = +6.67 \text{ D}$$
$$U_2 + P_2 = V_2$$
$$+6.67 - 20 = -13.33 \text{ D}$$